Prognostications With Applications to the British Pound

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Received: November 21, 2018 Accepted: May 23, 2019 Online Published: July 22, 2019
doi:10.5430/ijfr.v10n4p143 URL: https://doi.org/10.5430/ijfr.v10n4p143

Abstract

This paper scrutinizes several exchange rate models, considers the effectiveness of their predictive performance after applying both parametric and nonparametric techniques to them, and chooses the forecasting predictor with the smallest root mean square forecast error (RMSE). Equation (34) displays empirical evidence consistent with a better example of an exchange rate model, although none of the evidence gives us a completely satisfactory forecast. In the end, the models’ error correction versions will be fit so that plausible long-run elasticities can be imposed on each model’s fundamental variables.

Keywords: efficiency, exchange rate determination, exchange rate policy, forecasting, foreign exchange

1. Introduction

Most economic time series do not have an invariant mean because they alternate phases of relative stability with periods of greater volatility. For example, an initial review of time-series data, including but not limited to currency exchange rates implies that these data are heteroscedastic because of the absence of this constant mean and variance, as opposed to being homoscedastic because of the presence of a stochastic variable with a constant variance. For any series with such volatility, the unconditional variance could be constant even though it could also be unexpectedly large. Some variables’ trends can possess stochastic or deterministic characteristics, with the type of estimation and forecast done on these ingredients (namely, a deterministic or a stochastic trend) having a great deal of influence on the results of that time series.

One may illustrate multiple exchange rates’ behavior by graphing them, noting their fluctuations across time, and confirming any initial impressions through rigorous testing. For example, these series are not stationary because the sample means have a strong appearance of heteroscedasticity and do not appear to be constant. The absence of any specific trend in these series makes it difficult to prove the existence of a time-invariant mean, with the American dollar-to-British pound exchange rate not showing a tendency of either increasing or decreasing being but one example. The dollar had gone through long periods of appreciation followed by bouts of depreciation without reversion to the long-run average; this sort of "random walk" is quite representative of non-stationary time series.

Any shock to such a series shows great persistence, e.g., the dollar/pound exchange rate experienced a very sharp surge upward in 1980, remained at that level for about four years, and did not return to somewhat near its previous level for another five years. These series’ volatility is not constant and some of them correlate with other series. In fact, such series are called “conditionally heteroscedastic” if the unconditional (long-run) variance is constant but also having localized periods of relatively high variance. For instance, large shocks in the U.S. appear nearly simultaneously in Great Britain and Canada, although these co-movements can often be predicted because of the underlying forces affecting all nations’ economies, including that of the U.S.

The disturbance term’s variance is assumed to be constant in conventional econometric models, although the series alternates unusually volatile periods with spells of relative tranquility. Therefore, the assumption of a constant variance in such unconventional cases is incorrect. As an investor holding only one currency, however, one might wish to forecast both the exchange rate and its conditional variance over the life of the investment. The unconditional
variance - namely, its long-run forecast - would not be important were one to buy the asset at time period t and sell it at t+ 1. Taylor (1995) and Kallianiotis (1985) provide a survey and reviews of the literature on exchange rate economics and Chinn and Meese (1995) examine four structural exchange rate models’ performance.

This paper is set forth as follows. Different trend models are described in section 2. Other linear time-series models are presented in section 3 with multiequation time-series models defined in section 4. The empirical results are given in section 5, with a summary of the findings presented at the end of section 6.

2. Time-Series Trends

One way to predict a time series’ variance is to introduce an independent variable that explicitly helps forecast that series’ volatility. Consider the simplest case, for example, in which

$$s_{t+1} = (\epsilon_{t+1} X)_t$$  \hspace{1cm} (1)

Where \(s_{t+1}\) = the spot exchange rate or the variable of interest, \(\epsilon_{t+1}\) = a white-noise disturbance term with variance \(\sigma^2\), and \(X_t\) = an independent variable that can be observed at time period t. If \(X_t = X_{t-1} = X_{t-2} = \ldots = \text{a constant}\), then the \(\{s_t\}\) sequence is a standard white-noise process with a constant variance.

If the realizations of the \(\{X_t\}\) sequence are not all equal, then the variance of \(s_{t-1}\) that is conditional on the observable value of \(X_t\) is

$$\text{Var} (s_{t+1}(X_t)) = X_t^2 \sigma^2$$  \hspace{1cm} (2)

We can represent the general solution to a linear stochastic difference equation with these four components: \(s_t = \text{cyclical + irregular + seasonal + trend}\)

Exchange rate series do not have an obvious tendency to revert to any long-run mean. One goal of econometricians is forming well-defined stochastic difference equation models that can simulate trending variables’ behavior, with the trends noted for their permanent effects on time series. Because of the stationary nature of the irregular component of a series, its effects will diminish over time while the trending elements and their effects will persist in long-term forecasts.

2.1 Deterministic Trends

One of \(s_t\)’s basic characteristics, despite its short-term volatility, is its long-term growth pattern. In fact, \(s_t\) may have a well-defined long-term trend after all. According to the research of Pindyck and Rubinfeld (1981), Chatfield (1985), and Enders (1995), there are at least eight models that both define this deterministic trend and can be used in the extrapolation and forecasting of \(s_t\), and they are:

**Linear time trend:**

$$s_t = \alpha_0 + \alpha_1 t + \epsilon_t$$  \hspace{1cm} (3)

**Exponential growth curve:**

$$S_t = Ae^{rt}$$  \hspace{1cm} (4)

Or

$$\ln S_t = \ln A + rt + \epsilon_t$$  \hspace{1cm} (5)

Or

$$s_t = \beta_0 + \beta_1 t + \epsilon_t$$  \hspace{1cm} (6)

**Logarithmic or stochastic autoregressive trend (the only function that can be applied for exchange rates):**

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \epsilon_t$$  \hspace{1cm} (7)

**Quadratic trend:**

$$s_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \epsilon_t$$  \hspace{1cm} (8)

**Polynomial time trend:**

$$s_t = \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \ldots + \zeta_n t^n + \epsilon_t$$  \hspace{1cm} (9)

**Logarithmic growth curve:**

$$s_t = 1 / (\theta_0 + \theta_1 t + \theta_2 t^2) ; \theta_2 > 0$$  \hspace{1cm} (10)
Or its stochastic approximation:

\[
\frac{\Delta s_t}{s_{(t-1)}} = k_0 - k_1 s_{(t-1)} + \varepsilon_t
\]  

(11)

Sales saturation pattern:

\[
S_t = e^{\lambda_0 - (\lambda_1 / t)}
\]  

(12)

Or

\[
s_t = \lambda_0 - (\lambda_1 / t) + \varepsilon_t
\]  

(13)

Where \(S_t\) = the spot exchange rate, \(t\) = the time trend, and the lowercase letters are the natural logarithms of their uppercase counterparts.

2.2 Models of Stochastic Trend

One can increase the deterministic trend models with the lagged values of both the \(\{s_t\}\) and \(\{\varepsilon_t\}\) sequences. These equations thus become models with their own stochastic trends. These models are:

(i) The Random Walk Model

The random walk model, which is actually a special case of the AR(1) process, appears to mimic the exchange rates’ behavior as shown below. These time series neither revert to any given mean nor fluctuate over time.

\[
s_t = \alpha_0 + \alpha_1 s_{(t-1)} + \varepsilon_t
\]  

(14)

with \(\alpha_0 = 0\) and \(\alpha_1 = 1\), and where \(s_t - s_{(t-1)} = \Delta s_t = \varepsilon_t\), becomes

\[
s_t = s_{(t-1)} + \varepsilon_t
\]  

(15)

The conditional mean of \(s_{(t+\lambda)}\) for any \(\lambda > 0\) is

\[
E_t S_{(t+\lambda)} = S_t + E\sum_{i=1}^{\lambda} \varepsilon_{(t+i)} = s_t
\]  

(16)

with a time-dependent variance:

\[
var(s_t) = var(\varepsilon_t + \varepsilon_{(t-1)} + ... + \varepsilon_{-1}) = t\sigma^2
\]  

(17)

The random walk process is nonstationary because the variance is not constant. Therefore, as

\[t \to \infty\] and \(var(s_t) \to \infty\),

(18)

the forecast function will be:

\[
E_t s_{(t+\lambda)} = s_t
\]  

(19)

(ii) The Random Walk Plus Drift Model

The random walk plus drift model adds a constant term \(\alpha_0\) to the random walk model above such that \(s_t\) becomes deterministic in part and stochastic in part:

\[
s_t = s_{(t-1)} + \alpha_0 + \varepsilon_t
\]  

(20)

The general solution for \(s_t\) is:

\[
s_t = s_0 + \alpha_0 t + \sum (i=1)^t \varepsilon_i
\]  

(21)

and

\[
E_t s_{(t+\lambda)} = s_0 + \alpha_0 (t + \lambda)
\]  

(22)

The forecast function by \(\lambda\) periods yields:

\[
E_t s_{(t+\lambda)} = s_t + \alpha_0 \lambda
\]  

(23)

(iii) The Random Walk Plus Noise Model

The \(s_t\) is obtained here by adding a white-noise component to a stochastic trend:

\[
s_t = \mu_t + n_t
\]  

(24)

and

\[
\mu_t = \mu_{(t-1)} + \varepsilon_t
\]  

(25)

Where \(\{n_t\}\) is a white-noise process with variance \(\sigma_n^2\) and \(\varepsilon_t\) and \(n_t\) are both independently distributed for all \(t\), \(E(\varepsilon_t n_{(t-\lambda)}) = 0\) and the \(\{\mu_t\}\) sequence represents the stochastic trend. This model’s solution can be written:

\[
s_t = s_0 - n_0 + \sum (i=1)^t \varepsilon_i + n_t
\]  

(26)
The forecast function is:

\[ E_t S_{t+\lambda} = s_t - n_t \]  

(27)

(iv) The General Trend Plus Irregular Model

One can substitute the so-called “trend plus noise model” for equation (25) above:

\[ \mu_t = \mu_{t-1} + \alpha_0 + \varepsilon_t \]  

(28)

where \( \alpha_0 \) is a constant and \( \{\varepsilon_t\} \) is a white-noise process.

The solution is:

\[ s_t = s_0 - n_0 + \alpha_0 t + \sum_{i=1}^{t} \varepsilon_i + n_t \]  

(29)

Let A(L) be a polynomial in the lag operator L. It is possible to supplement a random walk plus drift process with the stationary noise process A(L) n_t and thus obtain the “general trend plus irregular model”:

\[ s_t = \mu_0 + \alpha_0 t + \sum_{i=1}^{t} \varepsilon_i + A(L) n_t \]  

(30)

(v) The Local Linear Trend Model

The local linear trend model is constructed by combining several random walk plus noise processes. Let \( \{\varepsilon_t\} \), \( \{n_t\} \), and\( \{u_t\} \) be three mutually uncorrelated white-noise processes. The equations for the local linear trend model are:

\[ s_t = \mu_t + n_t \]

\[ \mu_t = \mu_{t-1} + \alpha_t + \varepsilon_t \]  

(31)

\[ \alpha_t = \alpha_{t-1} + u_t \]

This is the most detailed of all the above models because the other processes are special cases of the local linear trend model, which consists of the noise term \( n_t \) and the stochastic trend term \( \mu_t \). What is most important about the model for this paper is that the change in its trend yields a random walk plus noise:

\[ \Delta \mu_t = \mu_t - \mu_{t-1} = \alpha_t + \varepsilon_t \]  

(32)

The forecast function of \( s_{t+\lambda} \) equals the current value of \( s_t \) minus the transitory component \( n_t \), added to \( \lambda \) multiplied by the slope of the trend term in \( t \):

\[ E_t s_{t+\lambda} = (s_t - n_t) + \lambda (\alpha_0 + u_{1} + u_{2} + \ldots + u_{t}) \]  

(33)

In the future, one may estimate these models and run different tests on both the error terms and the series as well as conclude with diagnostic and specification tests as a way of gauging the statistical specifications’ fitness and then compare the forecasting results obtained from the different models.

3. Some Linear Time-Series Models

In this section, stochastic processes are defined and some of their properties are discussed for use in forecasting, with a view toward developing models that explicate the movement of the time series \( s_t \). However, this will not be accomplished using explanatory variables per the regression model but by relating the series to its previous values and to a weighted sum of lagged and current random disturbances.

3.1 The Autoregressive (AR) Model

In the autoregressive process of order p, the current observation \( s_t \) is generated by a weighted average of past observations going back p time periods, together with the current time period t’s random disturbance. This process is defined as AR(p) and is written:

\[ s_t = \phi_1 s_{t-1} + \phi_2 s_{t-2} + \ldots + \phi_p s_{t-p} + \delta + \varepsilon_t \]  

(34)

\( \delta \) is a constant term which relates to the mean, or arithmetic average, of the stochastic process.

The first-order process AR(1) is:

\[ s_t = \phi_1 s_{t-1} + \delta + \varepsilon_t \]  

(35)

and its mean is:

\[ \mu = \delta / (1 - \phi_1) \]  

(36)

and is also stationary if \( \phi_1 < 1 \). (The random walk with drift is a first-order autoregressive process that is not stationary, however.)
4. Empirical Evidence

An analysis and summary of the empirical evidence for different models of foreign currency forecasting is included. The data given below are monthly from March 1973 through and including December 1994, are coming from Main Economic Indicators of the OECD (the Organization for Economic Cooperation and Development) and International Financial Statistics of the IMF (the International Monetary Fund), and they have been applied for the United Kingdom (U.K.). The exchange rate is defined as the U.S. dollar per unit of the British pound, with direct quotes for the dollar; the lowercase letters denote the variables’ natural logarithms and an asterisk denotes the corresponding variable for the U.K.

The first equations estimated are the deterministic trend models in equations (3), (6), (8), (9), (11), and (13). The results appear in Table 1 below and indicate that the exchange rate forecast cannot be supported by models of this type. The second group of equations, encompassing (15) and (20) in Table 2, is for the stochastic trend model and shows that this alternative model is noticeably better at both interpreting the data and forecasting the exchange rate. The final model is of a linear time-series, namely, the autoregressive (AR) model of equation (34) shown in Table 3, but its results are also mediocre at best. One may infer that time-series models cannot be used in the forecasting of foreign currency exchange rates with a great degree of confidence for models with such relatively high volatility.

Table 1. Deterministic trends

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Equation</th>
<th>Coefficients</th>
<th>t-values</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Linear time trend, eq. (3):</td>
<td>$S_t = \alpha_0 + \alpha_1 t + \varepsilon_t$</td>
<td>$\alpha_0 = 223.426^{***}$</td>
<td>(3.750)</td>
<td></td>
</tr>
<tr>
<td>(ii) Exponential Growth Curve, eq. (6):</td>
<td>$S_t = \beta_0 + \beta_1 t + \varepsilon_t$</td>
<td>$\beta_0 = 5.405^{***}$</td>
<td>(.021)</td>
<td></td>
</tr>
<tr>
<td>(iii) Quadratic Trend, eq. (8):</td>
<td>$S_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \varepsilon_t$</td>
<td>$\delta_0 = 5.633^{***}$</td>
<td>(.036)</td>
<td></td>
</tr>
<tr>
<td>(iv) Polynomial time trend, eq. (9):</td>
<td>$S_t = \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \ldots + \zeta_n t^n + \varepsilon_t$</td>
<td>$\zeta_0 = 3.621^{***}$</td>
<td>(.487)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_1 = -.005^{***}$</td>
<td>(.0005)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_2 = .189^{***}$</td>
<td>(.035)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_3 = .0001^{***}$</td>
<td>(1.4-05)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_4 = -.9-9-07^{***}$</td>
<td>(1.1-07)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_5 = 4.7-09^{***}$</td>
<td>(4.7-10)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\zeta_6 = -1.1-11^{***}$</td>
<td>(1.1-12)</td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\zeta_t & \quad 1.14^{***} \\
& \quad (9.9-16) \\
R^2 & \quad .468 \\
D-W & \quad .065 \\
SSR & \quad 4.478 \\
F & \quad 111.35 \\
RMSE & \quad .1323 \\
\end{align*}
\]

\[
\begin{align*}
D-W & \quad .152 \\
SSR & \quad 1.850 \\
F & \quad 125.84 \\
RMSE & \quad .0850 \\
\end{align*}
\]

(v) Stochastic approximation, eq. (11):  
\[
\Delta s_t = k_0 + k_1 s_{t-1} + \varepsilon_t 
\]

(vi) Sales Saturation Pattern, eq. (13):  
\[
s_t = \lambda_0 - (\lambda_1 t) + \varepsilon_t 
\]

\[
\begin{align*}
k_0 & \quad .114^* \\
& \quad (.060) \\
\lambda_0 & \quad 5.025^{***} \\
\end{align*}
\]

\[
\begin{align*}
k_1 & \quad -.022^* \\
& \quad (.011) \\
\lambda_1 & \quad 16.668^{***} \\
\end{align*}
\]

\[
\begin{align*}
R^2 & \quad .015 \\
D-W & \quad .433 \\
SSR & \quad 1.772 \\
F & \quad 3.796 \\
RMSE & \quad .0333 \\
\end{align*}
\]

Notes: \( s_t \) = the spot exchange rate, \( s_t = \ln(s_t) \), \( t \) = time, \( D-W \) = the Durbin-Watson statistic, \( SSR \) = sum of squares residuals, \( RMSE \) = root mean square error, data from 1973.03 to 1994.06, \( * * * \) = significant at the 1% level, \( * * \) = significant at the 5% level, \( * \) = significant at the 10% level, \( \Delta \) = change of the variable.

### Table 2. Stochastic trends

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_t = s_{t-1} + \varepsilon_t )</td>
<td>( s_t = \alpha s_{t-1} + \alpha_0 + \varepsilon_t )</td>
</tr>
<tr>
<td>( s_{t-1} )</td>
<td>.100^{***}</td>
</tr>
<tr>
<td>( \quad )</td>
<td>(.0004)</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>.114^*</td>
</tr>
<tr>
<td>( \quad )</td>
<td>(.060)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>.978^{***}</td>
</tr>
<tr>
<td>( \quad )</td>
<td>(.011)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
R^2 & \quad .966 \\
D-W & \quad 1.785 \\
SSR & \quad .289 \\
L(.) & \quad 505.31 \\
RMSE & \quad .0336 \\
\end{align*}
\]

Notes: See the previous table. \( L(.) \) = logarithm of likelihood function.

### Table 3. Linear time-series models

The Autoregressive (AR) Model, eq. (34):  
\[
s_t = \phi_1 s_{t-1} + \phi_2 s_{t-2} + ... + \phi_p s_{t-p} + \delta + \varepsilon_t 
\]

\[
\begin{align*}
\delta & \quad 5.139^{***} \\
& \quad (.078) \\
\phi_1 & \quad 1.093^{***} \\
& \quad (.063) \\
\end{align*}
\]
5. Summary

This paper examines the predictive performance of several foreign exchange rate forecast models, namely, linear time-series, the vector autoregression model, and various time-series trends. For every such forecast model, we calculate its root mean square forecast error (RMSE) as

\[ \text{RMSE} = \frac{1}{n} \sqrt{\sum_{t=1}^{n} (A_t - F_t)^2} \]

Where \( n \) = the number of observations, \( A \) = the actual value of the dependent variable, and \( F \) = the forecast value. The forecast model with the smallest RMSE is the predictor chosen as part of exchange rate forecasting. Movements in exchange rates may result from either a parametric change in the above determinants or an artificial intervention by governments.

References


