

# A Solution of GM(1,1) based on EXCEL

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## Abstract

The air cargo throughput forecasting can be used by five methods, which are the expert-determined method, analogy method, the trend extrapolation, the econometric methods, and the market analysis. In this paper, a new forecasting method, grey system model is put forward in the case of less information to get high precision of prediction results, and a GM(1,1) model is created about air cargo throughput based on Microsoft Office Excel and VBA Programming for the technology platform. Examples are given to show the model is effective and reasonable with higher forecasting accuracy and only 0.3% errors, which proves that the solution of GM(1,1) based on EXCEL can reduce the technical requirements for the staff, as EXCEL can be used to solve the problem without much programming background, and staff can also easily master and utilize GM (1, 1) model.

**Keywords:** GM(1,1), Air cargo throughput, Microsoft office excel

## 1. Generation of Sequence Operator and Gray Column Sequence

Gray system seeks its variation through the collation of the original data, which is a way of looking for the reality of the law of the data according to the data, known as the gray sequence generation. Any gray sequences can weaken its randomness through certain generation, showing its regularity.

### 1.1 Sequence Operators

Suppose

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \quad (1)$$

acts as the true sequence of system, the observed system behavior data sequence is:

$$X = (x(1), x(2), \dots, x(n)) = (x^{(0)}(1) + \varepsilon_1, x^{(0)}(2) + \varepsilon_2, \dots, x^{(0)}(n) + \varepsilon_n) = X^{(0)} + \varepsilon \quad (2)$$

Item  $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$  is one of the impact disturbance term, then X is the impact disturbance sequence.

If X is the system behavior based data sequence, D is the operator acting on the X. X, then, obtains the sequence recorded as

$$XD = (x(1)d, x(2)d, \dots, x(n)d) \quad (3)$$

D is called the sequence operator, and XD the first-order operator sequence. If  $D_1, D_2, D_3$  are the sequence operators,  $D_1 D_2$  is called the second-order operator, and  $X D_1 D_2 = (x(1)d_1 d_2, x(2)d_1 d_2, \dots, x(n)d_1 d_2)$  called the second-order operator sequence, and so on.

### 1.2 Generation of Mean

The generation of mean is a method commonly used to generate new data structure, filling the old hole sequence and generate the new sequence. Set sequence:

$$X = (x(1), x(2), \dots, x(k), x(k+1), \dots, x(n)) \quad (4)$$

$x(k)$  and  $x(k+1)$  is a pair of adjacent values with  $X$ ,  $x(k)$  is known as the anterior value,  $x(k+1)$  is called the after value. If  $X(N)$  is the new information, to the arbitrary  $k \leq n-1$ ,  $x(k)$  is the old information.

Set sequence:

$$X = (x(1), x(2), \dots, x(n)) \quad (5)$$

If:

$$x^*(k) = 0.5x(k) + 0.5x(k-1) \quad (6)$$

$x^*(k)$  is referred to as close to the mean generation number, the mean generated by a number of poses known as close to the mean of the sequence generated sequence.

Modeling at GM, the commonly used information close to the mean generation, is the constructing method of new sequence based on the original sequence.

## 2. Gray Model Mechanism and Five-step Modeling Method

### 2.1 Data Arranging Method

Through the collation of original data, looking for the number of laws, which can be divided into three categories:

#### (1) AGO

New data series are obtained through the various time of data cumulation one by one. The number cumulated before is the original series, while the number cumulated after is the generation of cumulative series. The basic relationship is as the following:

Recorded  $x^{(0)}$  as original series:

$$\begin{aligned} X^{(0)} &= (x^{(0)}(k), \quad k = 1, 2, \dots, n) \\ &= (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \end{aligned} \quad (7)$$

Recorded  $x^{(1)}$  as generating series:

$$\begin{aligned} X^{(1)} &= (x^{(1)}(k), \quad k = 1, 2, \dots, n) \\ &= (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \end{aligned} \quad (8)$$

If  $x^{(0)}$  and  $x^{(1)}$  meet the following relationship:

$$x^{(1)}(k) = \sum_{i=0}^k x^{(0)}(i) \quad (9)$$

It is named the first AGO.

#### (2) Inverse accumulated generating

Cumulated subtraction of the anterior and after data generates the inverse operation. Inverse accumulated generating can revert the cumulated generating into non-generated series.

#### (3) Mapping generating

Other generation methods besides accumulated generating and inverse accumulated generating

## 2.2 Modeling Building

### 2.2.1 The Mechanism of Modeling

- (1) Processing the original data into generated data.
- (2) Revise the residuals (the minus between model calculated values and the actual value), and set up difference differential equation model.
- (3) Analysis based on the correlation convergence.
- (4) Obtain the data of GM model through the inverse way of reduction.

### 2.2.2 Adoption of the "five-step modeling (system qualitative analysis, factor analysis, a preliminary quantification, dynamic quantization, Optimize)"

The establishment of a difference differential equation model, GM (1, 1) prediction model.

The basic formula is:

$$x^{(0)} = x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n) \quad (10)$$

Once AGO:

$$x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m) \quad (11)$$

$$\begin{aligned} x^{(1)} &= (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \\ &= (x^{(0)}(1), x^{(0)}(1) + x^{(0)}(2), \dots, x^{(0)}(n-1) + x^{(0)}(n)) \end{aligned} \quad (12)$$

Albino equation  $x^{(1)}$  can be set up:

$$dx^{(1)} / dt + ax^{(1)} = U \quad (13)$$

That is the GM (1,1) model. The solution of the equation is:

$$x^{(1)}(k+1) = (x^{(1)}(1) - \frac{U}{a})e^{-ak} + \frac{U}{a} \quad (14)$$

## 4. Application of Gray Prediction at Air Cargo throughout in the Asia-Pacific Logistics

### 4.1 The Selection of Characterization of the Logistics Demand Index

Logistics demand is defined as, in a certain period of time and regional scope, the sum of acting demand including transport, packaging, inventory, loading and unloading, distribution, circulation, as well as information processing in the time and space because of the production or consumption.. The index system measurement of logistics demand has physical volume of the system (such as cargo, freight, inventory, processing capacity, etc.) and value systems (such as logistics costs, logistics revenue, value-added supply chain, etc.). Because of the lack of necessary statistical data, after application, one system can only be based on the prediction by the expertises according to their experience of the logistics demand. In order to quantify the demand for logistics research and to get the availability of data, a system of physical volume of cargo or freight is adopted to characterize the logistics demand. These two indicators are the result of transport activity. Although the demand for transportation is only the part of the logistics demand, and can not represent the whole process of the logistics service, the transportation is the basic activity throughout the whole logistics process. In addition, the traffic inevitably decides the relevant volume of the logistics industry (such as handling, loading and unloading, packaging, etc.). Therefore, cargo or freight, to a certain extent, can show the logistics demand, reflecting the changes in law, and it is feasible to use the historical data of freight or cargo volume to forecast the logistics demand and modeling.

In order to constitute an interval series, the model can be predicted based on the data of cargo throughput in the period (In thousands of tons)

Table 1. air cargo throughput at Pudong Airport

No	1	2	3	4	5
Cargo throughput	11.89	16.42	18.57	21.68	25.59

From the table above, a growth in logistics demand can be seen. But how will the logistics demand develop in the future is still a problem of the factors related to the change of logistics demand. Logistics demand is a gray system of "less data, incomplete information". There is no doubt that it is an effective tool to predict the gray system theory.

#### 4.2 The Implementation and Testing

The model is established to predict according to the data of cargo throughput (In 100,000 tons). Input and operate the data in EXCEL as follows: fill the cargo throughput data in C6: C10;

Set up GM (1,1) model, the steps as follows:

(1) Copy C6 to D6 and F6, D7, enter "D7 = D6 + C7", filled D8: D10 with fill handle, then we get cumulative sequence: 11.89; 28.31; 46.88; 68.56; 94.15.

(2) Fill type "= -0.5 \* (D6 + C7)" in C17, ,enter one in D17, filled C17: D20 down with handle;

(3) Select regions H17: K18, enter "= TRANSPOSE (C17: D20)", and hold down the Ctrl + Shift + Enter ,then the values can be got as follows:

$$B^T = \begin{pmatrix} -20.100 & -37.595 & -57.720 & -81.355 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

(4) Select regions G22: H23, enter "= MMULT (H17: K18, C17: D20)", and hold down the Ctrl + Shift + Enter, then the values as follows:

$$B^T B = \begin{pmatrix} 11767.62845 & -196.77 \\ -196.77 & 4 \end{pmatrix}$$

(5) Select regions G25: H26, enter "= MINVERSE (G22: H23)", and hold down the Ctrl + Shift + Enter, then the values as follows:

$$(B^T B)^{-1} = \begin{pmatrix} 11767.62845 & -196.77 \\ -196.77 & 4 \end{pmatrix}^{-1} = \begin{pmatrix} 0.000478923 & 0.023559398 \\ 0.023559398 & 1.408945698 \end{pmatrix}$$

(6) Select regions C7: C10, selected regions H29: K30, enter "= MMULT (G25: H26, H17: K18)", then we get

$$(B^T B)^{-1} B^T$$

(7) Select regions G35: G36, enter "= MMULT (H29: K30, C7: C10)", then we get the following values  $\alpha$  and  $U$ , that is,  $\alpha = -0.150788841$ ,  $U = 13.14731993$

(8) Enter in E7

"= (\$ E \$ 6 - \$ G \$ 36 / \$ G \$ 35) \* EXP (- \$ G \$ 35 \* A6) + \$ G \$ 36 / \$ G \$ 35", filled "E8: E10" down with fill handle, through matrix computation, enter the appropriate formula, press key combination "Ctrl + Shift + Enter",

$x_{k+1}^{(1)} = (x_0^{(1)} - \frac{U}{\alpha})e^{-\alpha k} + \frac{U}{\alpha}$  the value of  $x_{k+1}^{(1)} = (x_0^{(1)} - \frac{U}{\alpha})e^{-\alpha k} + \frac{U}{\alpha}$  is obtained as follows: 28.02, 46.77, 68.57, 93.92.

(9) Fill type "= E7-E6" in F7, and fill "F8: F10" down with fill handle to draw a few numerical prediction as follows:

Table 2. The deviation comparison of Cargo prediction

NO.	Cargo throughput	Series prediction	Residual sequence	Relative error sequence
1	11.89	11.89	0.0000	0
2	16.42	16.13	0.2946	0.017940086
3	18.57	18.75	-0.1799	-0.009685211
4	21.68	21.80	-0.1214	-0.005600273
5	25.59	25.35	0.2404	0.009393585

Part of the calculation of test accuracy, the basic computing of EXCEL, shall not be repeated here. As can be seen from the table above, the maximum relative error of gray prediction is 0.2946, so the fitting is in good condition. The precision test of Cargo, mail throughput: The relative deviation in average  $\Delta = 0.003012047 < 0.01$ , is in a high level of accuracy; the variance ratio  $C = 0.040933653 < 0.35$ , is in a high level of accuracy; small deviation probability  $p = 1.0 > 0.95$ , is in a high level of accuracy.

Table 3. Level Table of Model accuracy

P	C	Accuracy Level
$P > 0.95$	$C < 0.35$	Good (1)
$0.8 < P \leq 0.95$	$0.35 \leq C < 0.5$	Qualified (2)
$0.7 < P \leq 0.8$	$0.5 \leq C < 0.65$	Barely qualified (3)
$P \leq 0.7$	$0.65 \leq C$	Fail (4)

According to this model, air cargo throughput can be calculated. Through the use of EXCEL, a predictive value of the next five years can be generated. In E11, handle is used to fill "E11: E15" down, then through matrix calculation, enter the corresponding formula, press key combination "Ctrl + Shift + Enter" values, then the values of  $x_{k+1}^{(1)} = (x_0^{(1)} - \frac{U}{a})e^{-ak} + \frac{U}{a}$  can be got as follows: 123.39, 157.66, 197.51, 243.85, 297.73. Fill "F11: F15" with

handle down, then a few numerical prediction can be drawn as follows:

Table 4. Forecast for the throughput of air cargo in the next five years

NO.	Series prediction
6	29.48
7	34.27
8	39.85
9	46.34
10	53.88

### 5. Conclusion

As an important part of China logistics, modern air logistics is of great importance to the development of the industry and can give much access to the new opportunities. Forecast aviation throughput is an important field in airport project with great feasibility for research. Furthermore, the airport is the fundamental basis for development planning. Researches need to be done to forecast the future of the air logistics. According to the research, the construction of the airport facilities, size and scale, and a reasonable construction phases can be confirmed. In the prediction of aviation business of the civil airport, the previous cargo throughput of the airport is known, while the future cargo throughput is unknown, so the cargo throughput of the airport is a gray system. In this paper, the prediction model of air cargo throughput is set up using the gray system, and the prediction method of gray model is put forward. Besides, an effective algorithm through the gray prediction model is established by EXCEL. Examples are given to show the model is effective, reasonable and have higher forecasting accuracy, and the errors are only 0.3%, showing that EXCEL tool can be utilized to solve the problem, even if without understanding programming, and GM (1, 1) model is easy for the staff to master and practice. So the solution of GM(1,1) based on EXCEL will reduce the technical requirements for the staff.

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