

Assessing Performance of Liquidity Adjusted Value-at-Risk Models

Vandana Rao Daka¹ & Sankarshan Basu¹

¹ Indian Institute of Management Bangalore, Bangalore, India

Correspondence: Vandana Rao Daka, Indian Institute of Management Bangalore (IIMB), Bangalore, India.

Received: September 1, 2016

Accepted: September 22, 2016

Online Published: October 8, 2016

doi:10.5430/ijfr.v7n5p87

URL: <http://dx.doi.org/10.5430/ijfr.v7n5p87>

Abstract

In this paper, a portfolio-level Liquidity Adjusted Value at Risk model is formulated by using the adapted approach based on the Cornish-Fisher expansion technique to account for non-normality in liquidity risk. Most models ignore the fact that liquidity costs which measure market liquidity are non-normally distributed and this leads to a severe underestimation of the total risk. The Cornish-Fisher expansion technique, as proposed by prior studies is used for correcting the percentiles of a standard normal distribution for non-normality and is simple to implement in practice. The empirical evidence obtained in this study shows that accounting for non-normality at portfolio level and using the modified approach produces much more accurate results than alternative risk estimation methodologies. The model is tested using emerging markets' data as research on liquidity that primarily focuses on emerging markets yield particularly powerful tests and useful independent evidence since liquidity premium is an important feature of these data.

Keywords: value at risk, liquidity costs, emerging markets

1. Introduction

Large and random security price movements during financial crises cause liquidity gaps and most hedging strategies tend to fail when these crises occur. The financial crisis and the subsequent global recession of 2008-2012 have demonstrated how “a complete evaporation of liquidity” (Note 1) can cause the collapse of many financial institutions. Existing literature shows that investors should worry about a security's performance and tradability both in market downturns and when liquidity “dries up” (Amihud 2002, Chordia et al. 2001, Acharya and Pedersen 2005, Bekaert et al. 2007). There are many alternative measures of liquidity in the literature such as quoted bid-ask spreads, effective bid-ask spreads, turnover, the ratio of absolute returns-to-volume, adverse-selection and market-making cost components of price impact (Korajczyk and Sadka 2008).

Prior studies have analyzed the importance of liquidity risk using a comprehensive liquidity measure in a Value-at-Risk (VaR) framework (Jarrow and Subramaniam 1997, Bangia et al. 2002, Angelidis and Benos 2006, Stange and Kaserer 2011). However, most LVaR models ignore the fact that liquidity costs, which measure market liquidity, are non-normally distributed displaying fat tails and skewness. Many studies show that the assumption of normally distributed returns is rejected for most financial time series, including those for individual stocks, stock indexes, exchange rates and precious metals. The argument of non-normality holds equally for liquidity costs. Stange and Kaserer (2008) analyze the distributional properties of liquidity costs and show that they are heavily skewed and fat-tailed. Ernst et al. (2012) suggest a parametric approach based on the Cornish-Fisher approximation to account for non-normality in liquidity risk.

The goal of this paper is to extend the concept of including liquidity measure in centralized risk management tools such as VaR in order to develop a portfolio LVaR (Liquidity adjusted Value-at-Risk) model. The univariate or instrument level methodology suggested by Ernst et al. (2012) is used to develop a portfolio level LVaR model. The data on Indian stocks is used for the empirical part of the analysis as research on liquidity that primarily focuses on emerging markets yield particularly powerful tests and useful independent evidence since the liquidity premium is an important feature of these data (Bekaert et al. 2007). In recent years, many financial institutions have seen growth in their emerging markets trading activity due to higher margins. A risk-adjusted view of performance in those markets should account for liquidity risk as it is usually found to be higher in emerging markets due to lower volumes.

The paper is organized as follows; Section 2 provides a comprehensive literature review, Section 3 discusses the research methodology, Section 4 describes the data, Section 5 discusses the empirical performance of the modified LVaR model at the portfolio level, Section 6 presents robustness checks and Section 7 concludes.

2. Literature Review

The risk that a given security or asset cannot be traded quickly enough in the market to prevent or minimize a loss is termed liquidity risk. The last decade has seen considerable amount of research work directed towards managing liquidity risk while pricing an option. According to Acharya and Pedersen (2005), liquidity is risky and has commonality: it varies over time both for individual stocks and for market as a whole. Their Liquidity –Adjusted Capital Asset Pricing Model provides a unified theoretical framework that explains the empirical findings that return sensitivity to market liquidity is priced (Pastor and Stambaugh, 2003), that average return is priced (Amihud and Mendelson, 1986) and that liquidity commoves with returns and predicts future returns (Amihud, 2002; Chordia et al., 2001; Bekaert et al., 2007). Said differently, the model implies that investors should worry about a security’s performance and tradability both in market downturns and when liquidity “dries up”. Brunnermeier and Pedersen (2009) provide a model that links an asset’s market liquidity and trader’s funding liquidity. The model explains empirically documented features that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to “flight to quality” and (v) co-moves with the market. Importantly, the model links a security’s market illiquidity and risk premium to its margin requirement (i.e. funding use) and the general shadow cost of funding.

There are many alternative measures of liquidity in the literature. Measures that have appeared in the literature include quoted bid-ask spreads, effective bid-ask spreads, turnover, the ratio of absolute returns-to-volume, and adverse-selection and market-making cost components of price impact. Korajczyk and Sadka (2008) estimate latent factor models of liquidity and a measure of global, across-measure systematic liquidity by estimating a latent factor model pooled across all measures. The results show that there is commonality, across assets, for each individual measure of liquidity and that these common factors are correlated across measures of liquidity. Return shocks are contemporaneously correlated with liquidity shocks and lead changes in liquidity. Additionally, shocks to liquidity tend to die out slowly over time.

Liquidity risk is neglected by widely used risk management measures such as VaR. Derivatives users generally calculate a VaR measure for their derivatives portfolio and by not taking into account the liquidity risk component; they underestimate the portfolio risk exposures. *VaR is an estimate of the maximum potential loss that may be incurred on a position for a given time horizon and a specified level of confidence.* Since the publication of the market-risk-management system RiskMetrics (Note 2) of JP Morgan in 1994, VaR has gained increasing acceptance and is now considered as industry’s standard tool to measure market risk. In calculating VaR, it is assumed that the positions concerned can be liquidated or hedged within a fixed and fairly short timeframe (in general one day to ten days), that the liquidation of positions will have no impact on the market and that the bid-ask spread will remain stable irrespective of the size of the position, in essence a perfect market is assumed. The price referred to is often the mid-price or the last known market price. However, the quoted market price cannot be used as a basis for valuating a portfolio that is to be sold on a less than perfectly liquid market: in practice, account must be taken of its orderly liquidation value or even its distress liquidation value.

Jarrow and Subramaniam (1997) were among the first to estimate liquidity-adjusted VaR (LVaR), taking account of the expected execution lag in closing a position and the market impact of prices being adversely effected by a quantity discount that varies with the size of the trade. The model requires three quantities which increase the loss level – namely a liquidity discount, the volatility of the liquidity discount and the volatility of the time horizon to liquidation. Whilst this model is robust and fairly easy to implement, estimating these quantities is by no means trivial. Indeed, some may only be determined empirically with the accompanying introduction of significant bias. Bangia et al. (2002) propose similar measures of LVaR, they classify the liquidity risk into two different categories: (i) the exogenous illiquidity that depends on the general conditions of the market and (ii) the endogenous illiquidity which relates the position of a trader with the bid-ask spread. By focusing on the exogenous risk, they construct an LVaR measure for both the underlying instrument and the bid-ask spread. Specifically, they adjust the VaR number for “fat” tails and for the variation of the bid-ask spread.

Hisata and Yamai (2000) propose a practical framework for the quantification of LVaR which incorporates the market liquidity of financial products. The framework incorporates the mechanism of the market impact caused by the investor’s own dealings through adjusting VaR according to the level of market liquidity and the scale of the investor’s position. In addition, they propose a closed-form solution for calculating LVaR as well as a method of estimating

portfolio LVaR. Angelidis and Benos (2006) relax the traditional, yet unrealistic, assumption of a perfect, frictionless financial market where investors can either buy or sell any amount of stock without causing significant price changes. They extend the work of Madhavan et al. (1997) (who argue that traded volume can explain price movements) and develop a liquidity VaR measure based on spread components. Under this framework, the liquidity risk is decomposed into its endogenous and exogenous components, thereby permitting an assessment of the liquidation risk of a specific position.

Stange and Kaserer (2011) analyze the importance of liquidity risk using a comprehensive liquidity measure, weighted spread, in a Value-at-Risk (VaR) framework. The weighted spread measure extracts liquidity costs by order size from the limit order book. Using a unique, representative data set of 160 German stocks over 5.5 years, they show that liquidity risk is an important risk component. Liquidity risk increases the total price risk by over 25%, even at 10-day horizons and for liquid blue chip stocks and especially in larger, yet realistic order sizes beyond €1 million. When correcting for liquidity risk, it is commonly assumed that liquidity risk can be simply added to price risk. The empirical results show that this is not correct, as the correlation between liquidity and price is non-perfect and total risk is thus overestimated. According to Ernst et al. (2012) liquidity costs, which measure market liquidity, are non-normally distributed, displaying fat tails and skewness. Most liquidity risk models either ignore this fact or use the historical distribution to empirically estimate worst losses. They suggest a parametric approach based on the Cornish-Fisher approximation to account for non-normality in liquidity risk. They show how to implement this methodology in a large sample of stocks and provide evidence that it produces much more accurate results than alternative empirical risk estimation.

3. Research Methodology

Value-at-Risk (VaR) is a number that represents the potential change in a portfolio's/asset's future value. This change is defined based on (1) the horizon over which the portfolio's change in value is measured and (2) the "degree of confidence" chosen by the risk manager. To compute the VaR of an asset over a 1-day horizon with $\alpha\%$ chance (confidence interval) that the actual loss in the asset's value does not exceed VaR estimate consists of the following steps:

Asset returns r_t are computed as the log difference of mid-prices $P_{mid,t}$ (the average of bid ask values of the asset at time t)

$$r_{t+1} = \ln[P_{mid,t+1}] - \ln[P_{mid,t}] = \ln\left[\frac{P_{mid,t+1}}{P_{mid,t}}\right]$$

The $\alpha\%$ worst case value assuming normal returns is

$$\hat{P}_{t+1} = P_{mid,t} e^{[\mu_r - z_\alpha(r)\sigma_r]}$$

Where

$$\begin{aligned}\hat{P}_{t+1} &= P_{mid,t} e^{\hat{r}_{t+1}} \\ \hat{r}_{t+1} &= \mu_r - z_\alpha(r)\sigma_r\end{aligned}$$

Assuming the return on this asset is distributed conditionally normal, the relative VaR estimate is

$$VaR = \frac{P_{mid,t} - \hat{P}_{t+1}}{P_{mid,t}} = \frac{P_{mid,t}(1 - e^{[\mu_r - z_\alpha(r)\sigma_r]})}{P_{mid,t}} = (1 - e^{[\mu_r - z_\alpha(r)\sigma_r]}) \quad (\text{Equation 1})$$

The above expressions for $\alpha\%$ worst case value (\hat{P}_{t+1}) and potential loss (relative VaR estimate) only consider the volatility of the mid-price, whereas on an average the bid-price is expected to be $\frac{1}{2}$ times average spread below that. Moreover, in unusual tail-event circumstances due to overall market conditions liquidity risk is defined in terms of a confidence interval or a tail probability. Bangia et al. (2002) define the exogenous cost of liquidity (COL) based on average spread plus a multiple of the spread volatility $\hat{z}_\alpha(S)\sigma_S$ to cover $\alpha\%$ of the spread situations

$$COL = \frac{1}{2}[P_{mid,t}(\mu_S + \hat{z}_\alpha(S)\sigma_S)]$$

The achievable transaction price $P_{TA,t+1}$ accounting for liquidity cost is

$$P_{TA,t+1} = P_{mid,t} e^{[\mu_r - z_\alpha(r)\sigma_r]} - COL = P_{mid,t} e^{[\mu_r - z_\alpha(r)\sigma_r]} - \frac{1}{2}[P_{mid,t}(\mu_S + \hat{z}_\alpha(S)\sigma_S)]$$

$$P_{TA,t+1} = P_{mid,t} \left(e^{r_{t+1}} - \frac{1}{2} e^{r_{t+1}} S_{t+1} \right)$$

Where

$$r_{t+1} = \mu_r - z_\alpha(r) \sigma_r$$

$$S_{t+1} = \mu_S + \hat{z}_\alpha(S) \sigma_S$$

Applying the simplification that $e^{r_{t+1}}$ is almost equal to 1, the price is

$$P_{TA,t+1} \approx P_{mid,t} \left(e^{r_{t+1}} - \frac{1}{2} S_{t+1} \right)$$

The relative Liquidity-adjusted VaR measure (assuming a normal distribution with mean μ_r as zero) according to Bangia et al. (2002) is

$$LVaR = \frac{P_{mid,t} - P_{TA,t+1}}{P_{mid,t}}$$

$$LVaR = \frac{P_{mid,t} - P_{mid,t} e^{r_{t+1}} \left(1 - \frac{1}{2} S_{t+1} \right)}{P_{mid,t}}$$

$$LVaR = 1 - e^{r_{t+1}} \left(1 - \frac{1}{2} S_{t+1} \right)$$

$$LVaR = 1 - e^{-z_\alpha(r) \sigma_r} + \frac{1}{2} (\mu_S + \hat{z}_\alpha(S) \sigma_S) \tag{Equation 2}$$

A normal distribution is fully described by its first two moments: mean and variance. Higher centralized moments like skewness and excess kurtosis are zero. However, if the distribution is non-Gaussian, higher moments will also determine loss probabilities. For this reason, it is not accurate to use standard percentiles of a normal distribution for the calculation of the LVaR of nonnormally distributed returns. Cornish and Fischer (1937) were the first to modify the standardized percentiles of a normal distribution in a manner that accounted for higher moments. They obtained explicit polynomial expansions for standardized percentiles of a general distribution in terms of its standardized moments and the corresponding percentiles of the standard normal distribution. Their procedure is commonly known as the Cornish-Fischer expansion. Using the first four moments (mean, variance, skewness and kurtosis), the Cornish-Fischer expansion approximating the α -percentile \tilde{z}_α of a standardized random variable is calculated as:

$$\tilde{z}_\alpha \approx z_\alpha + \frac{1}{6} (z_\alpha^2 - 1) * \gamma + \frac{1}{24} (z_\alpha^3 - 3z_\alpha) * \kappa - \frac{1}{36} (2z_\alpha^3 - 5z_\alpha) * \gamma^2 \tag{Equation 3}$$

Where z_α is the α -percentile of an $N(0,1)$ distribution, where γ denotes skewness and κ denotes the excess kurtosis of the random variable. The skewness of y is computed from historical data over n days as:

$$\gamma = \frac{1}{n} \sum_{t=1}^n \frac{(y_t - \bar{y})^3}{\hat{\sigma}^3} \tag{Equation 4}$$

With \bar{y} being the expected value and $\hat{\sigma}$ being the volatility of y . The excess kurtosis for y is:

$$\kappa = \frac{1}{n} \sum_{t=1}^n \frac{(y_t - \bar{y})^4}{\hat{\sigma}^4} - 3 \tag{Equation 5}$$

Ernst et al. (2012) propose an adapted model based on the Cornish Fisher expansion technique used to correct the percentiles of a standard normal distribution. They apply the Cornish-Fischer approximation \tilde{z}_α to the basic spread model of Bangia et al. (2002) to obtain the following modified LVaR estimate:

$$LVaR = 1 - e^{\mu_r - \tilde{z}_\alpha(r) \sigma_r} \left(1 - \frac{1}{2} (\mu_S + \tilde{z}_\alpha(S) \sigma_S) \right) \tag{Equation 6}$$

where $\tilde{z}_\alpha(r)$ is the percentile of the return distribution accounting for its skewness and kurtosis, $\tilde{z}_\alpha(S)$ is the corresponding spread distribution percentile. Ernst et al. (2012) use the methodology described above (Equation 6) to compute LVaR estimates at instrument level and simply take the mean of the LVaR estimates for the analysis of more

than one instrument. There is no explicit methodology suggested in their paper to compute a portfolio level LVaR model.

One approach for a full portfolio level treatment for liquidity risk is suggested in Bangia et al. (2002). They suggest computing the portfolio-level bid and ask series by taking the weighted sum of the bids and asks of the instruments. However, Bangia et al. (2002) assume that the returns are normally distributed while computing the portfolio LVaR estimates using this approach. Many studies (Stange and Kaserere 2011, Ernst et al. 2012) show that the assumption of normally distributed returns is rejected for most financial time series, including those for individual stocks, exchange rates, precious metals etc.

In this study, the portfolio level bid and ask series is computed by taking the weighted sum of the bids and asks of the instruments (suggested by Bangia et al. 2002) and this bid-ask data is used for calculating the portfolio-level estimate LVaR (Modified) using Equation 6 (discussed by Ernst et al. 2012). Therefore, this study discusses the approach for calculating a portfolio-level LVaR (Modified) measure by using the adapted model based on the Cornish-Fisher expansion technique used for correcting the percentiles of a standard normal distribution for non-normality.

4. Data Description

The required price and bid-ask spread data of the stocks is obtained from the database Datastream for the time period from January 2010 to December 2014. Table 1 contains the exact description of the sample portfolios used for the analysis. Indian stocks belonging to diverse sectors are selected based on data availability during the analysis period. Descriptive statistics of relative bid-ask spreads for the stocks in the Nifty portfolio are presented in Table 2. The analysis for all the portfolios is included in the next section.

Table 1. Compositions of equally-weighted portfolios for analysis

Nifty	Infra	Service	Midcap	Smallcap
Bajaj Auto	JSW Energy	Infosys	Apollo Hospitals	Bombay Dyeing
Cipla	Crompton Greaves	Adani Ports	DLF	Escorts
ITC	Tata Communications	Axis Bank	Jindal Steel	Chambal Fertilizers
Gail	IRB Infra.	Bharti Airtel	SUN TV	Gujarat Fluorochemicals

The relative bid-ask is found via formula,

$$Relative\ bid - ask\ spread = 2 * \frac{(ask - bid)}{ask + bid} \quad (\text{Equation 7})$$

Table 2. Descriptive statistics of relative bid-ask spreads calculation using Equation 7 (in percent)

	2010	2011	2012	2013	2014
BAJAJ AUTO					
Mean	0.113697	0.120219	0.099602	0.11798251	0.10985539
Standard deviation	0.093527	0.112087	0.084476	0.11388243	0.08982457
CIPLA					
Mean	0.098872	0.09883	0.08406	0.08636526	0.0887063
Standard deviation	0.088175	0.082422	0.067186	0.06988485	0.07172833
ITC					
Mean	0.078364	0.06339	0.066908	0.06300317	0.06114429
Standard deviation	0.059586	0.049315	0.052912	0.05235501	0.05165432

GAIL					
Mean	0.109129	0.133158	0.129201	0.14582128	0.13523665
Standard deviation	0.097361	0.114772	0.105513	0.13524092	0.11173876
Observations per firm	247	247	247	248	243

Table 2 shows that ITC is the most liquid stock with the smallest spread and GAIL is the least liquid stock with the largest spread for the time period from 2010 to 2014. The spread volatility values show that not only is the spread lowest for ITC but it also varied considerably less over time compared to the other stocks.

5. Empirical Performance

In this section, the risk estimates for the individual stocks are computed first using measures suggested by existing research to check whether the results obtained using emerging markets' data are consistent with the prior theory. Then the empirical estimates for the portfolio are computed using the modified LVaR model.

Conforming to the standard Basel framework, risk is estimated using a one-day horizon and a 99% confidence level. The values of relative spread means and return means required for the LVaR model (refer Equation 6) are estimated using a twenty day rolling procedure.

One day asset returns at time t are calculated as the log difference of mid-prices:

$$r_{t+1} = \ln[P_{mid,t+1}] - \ln[P_{mid,t}] = \ln \left[\frac{P_{mid,t+1}}{P_{mid,t}} \right] \quad (\text{Equation 8})$$

Volatilities of relative spread (Equation 7) and return (Equation 8) are also calculated rolling over twenty days. Volatility clustering is accounted for using a common exponential weighted moving average method with a weight δ of 0.94 as:

$$\sigma_t^2 = (1 - \delta) \sum_{i=1}^{20} \delta^{i-1} r_{t-i}^2 + \delta^{20} r_{t-20}^2 \quad (\text{Equation 9})$$

Skewness (Equation 4) and excess kurtosis (Equation 5) are calculated as 500-day rolling estimates. The long estimation horizon is chosen as the estimates are heavily influenced by outliers. However, to keep the sample as large as possible and to include the first two years in the results period, shorter rolling windows in the increasing order of 20, 50, 100 & 250-day are included at the beginning of the sample. Skewness and excess kurtosis estimates for Spread and return are presented in Table 3.

Table 3. Relative Spread & Return moment estimates

	(a) Spread moment estimates			
	BAJAJ AUTO	CIPLA	ITC	GAIL
Skewness				
Mean	1.693839209	1.822292261	2.013397495	1.772488258
Median	1.740354438	1.679014116	2.106195292	1.800388092
Standard deviation	0.278628996	0.384730719	0.384833247	0.555467708
Kurtosis				
Mean	3.647316173	4.922664444	6.690311755	5.595536253
Median	3.890146269	3.857497105	6.814681255	4.302191846
Standard deviation	1.441803241	2.462479477	2.661017741	4.253710592

(b) Return moment estimates				
	BAJAJ AUTO	CIPLA	ITC	GAIL
Skewness				
Mean	0.126332404	0.070600171	-0.05665188	0.045883306
Median	0.037823072	0.081129108	-0.16960306	-0.042300135
Standard deviation	0.25063248	0.281448566	0.423730486	0.262188642
Kurtosis				
Mean	0.851454442	1.674145	1.882035737	0.485880215
Median	0.902888808	1.424672729	1.858183712	0.337319939
Standard deviation	0.3489489	0.915809932	0.977181887	0.549955371

Empirical 99% percentile estimate of $\hat{z}_\alpha(S)$ shown in Table 4 are calculated according to the Bangia et al. (2002) framework as:

$$\hat{z}_\alpha = (\hat{S}_\alpha - \mu_S) / \sigma_S \quad (\text{Equation 10})$$

where \hat{S}_α is the percentile spread of the past twenty-day historical distribution and μ_S and σ_S are mean and volatility of the relative spread.

Table 4. Empirical percentile estimates for the Bangia model

	BAJAJ AUTO	CIPLA	ITC	GAIL
Mean	1.646200565	1.562148885	1.555050208	1.621059388
Median	1.600283939	1.504682854	1.477278938	1.595656437
Standard deviation	0.460316135	0.474548719	0.510702042	0.486299454

Using the first four moments (mean, variance, skewness and kurtosis), the percentiles based on the Cornish-Fisher approximation are calculated for relative spreads and returns using Equation 3 (Table 5).

Table 5. Cornish-Fischer percentile estimates – Spread & Return

	BAJAJ AUTO	CIPLA	ITC	GAIL
Spread				
Mean	3.314796833	3.510807394	3.788462333	3.638404654
Median	3.379408608	3.399985274	3.801174484	3.437969448
Standard deviation	0.234776953	0.323706586	0.44515262	0.738801237
Return				
Mean	2.588458345	2.737654317	2.655693383	2.446938434
Median	2.557345522	2.76865779	2.500223997	2.380306519
Standard deviation	0.153474096	0.192993233	0.37992527	0.202782626

Table 6 shows empirical risk estimates for VaR or Price risk (Equation 1), LVaR measure (Equation 2) according to Bangia et al. (2002) and the LVaR measure suggested by Ernst et al. (2012) methodology (Equation 6). The LVaR

measure provides the highest risk estimates suggesting that neglecting liquidity risk or the assumption of normally distributed returns leads to underestimation of the total risk of an asset. Since, ITC is the most liquid stock with the smallest spread and GAIL is the least liquid stock with the largest spread therefore as expected GAIL has the highest risk estimate and ITC has the lowest.

Table 6. Risk estimates for individual stocks

%	BAJAJ AUTO	CIPLA	ITC	GAIL
Price risk				
Mean	3.632656699	3.37084671	3.258682793	3.70610231
Median	3.467297221	3.186048101	3.106862453	3.549050878
Standard dev.	1.081814956	1.04598625	1.032842813	1.050924702
LVaR (Bangia et al.)				
Mean	3.803240547	3.504543781	3.354800877	3.904938768
Median	3.633192289	3.327872271	3.192487517	3.756645786
Standard dev.	1.087671406	1.05800485	1.034768181	1.060040644
LVaR (Ernst et al.)				
Mean	4.221186275	4.139432607	3.80161315	4.254589843
Median	4.03003585	3.943251974	3.612603736	4.107947502
Standard dev.	1.255921982	1.300857475	1.305408989	1.21632208

In order to compute the portfolio-level risk estimates, an equally-weighted portfolio is constructed using the stocks Bajaj Auto, Cipla, ITC and Gail. The portfolio level bid-ask series is computed by taking the equally weighted sum of the bids and asks of the instruments. Table 7 shows empirical estimates for the portfolio using equations 1-6. The portfolio LVaR (Modified) measure is calculated using the approach described in section 3 (Research Methodology). The portfolio level bid-ask series is computed by taking the weighted sum of the bids and asks of the instruments and this series is used to calculate LVaR (modified) measure. According to Table 7, the portfolio LVaR (Modified) measure provides the highest risk estimates, showing that neglecting liquidity risk or assuming that the returns are normally distributed leads to a severe underestimation of the total risk. The portfolio-level analysis is repeated using distinct portfolios (refer Table 1) and the results are presented in Tables 8, 9, 10 & 11. The results remain the same.

Table 7. Portfolio risk estimates (weights: Bajaj Auto = .25, Cipla = .25, ITC = .25 & GAIL = .25)

	Relative Spread	Return	Skewness(S)	Kurtosis(S)	Skewness (R)	Kurtosis (R)
Mean	0.107504734	0.000659609	1.467476417	2.71448549	0.054965396	0.569078752
Median	0.092079206	0.000591684	1.486144595	2.688464986	0.002645487	0.556628963
Std Dev.	0.066141839	0.012148188	0.291730956	1.082309236	0.192383672	0.244441789
	z-alpha(Bangia)	z-cornish(S)	z-cornish (R)	Price Risk (%)	LVaR (Bangia) %	LVaR (Modified) %
Mean	1.222216147	3.196860544	2.48462352	2.681242886	2.809134596	3.026726895
Median	1.195095316	3.216251046	2.455297707	2.564681479	2.69314726	2.907314097
Std Dev.	0.351456225	0.201711476	0.16309667	0.765116261	0.772016994	0.860140693

Table 8. Portfolio risk estimates - Infra (weights: JSW Energy = .25, Crompton Greaves = .25, Tata Communications = .25 & IRB Infra. = .25)

	Relative Spread	Return	Skewness(S)	Kurtosis(S)	Skewness (R)	Kurtosis (R)
Mean	0.157935569	1.41699E-05	5.355810542	71.75344423	-0.248326109	0.469428191
Median	0.142673258	0.000426268	7.519791353	106.3839229	-0.288795359	0.351296393
Std Dev.	0.100376706	0.018994293	3.525948658	58.09914333	0.188124925	0.561464923
	z-alpha(Bangia)	z-cornish(S)	z-cornish (R)	Price Risk (%)	LVaR (Bangia) %	LVaR (Modified) %
Mean	0.962803869	7.565285392	2.217071326	4.103538767	4.274740198	4.626610064
Median	0.826826105	7.052219911	2.147985599	3.896207647	4.03519867	4.446329983
Std Dev.	0.51004316	3.942126764	0.190830865	1.322265372	1.356173616	1.580766816

Table 9. Portfolio risk estimates – Service (weights: Infosys = .25, Adani Ports = .25, Axis Bank = .25 & Bharti Airtel = .25)

	Relative Spread	Return	Skewness(S)	Kurtosis(S)	Skewness (R)	Kurtosis (R)
Mean	0.0741	0.000435111	1.714800637	6.503649642	-0.653438834	5.888855817
Median	0.06701	0.000322182	1.710293617	7.150128066	-0.400893165	1.650917704
Std Dev.	0.03893	0.013166059	0.689939776	5.484808131	0.547232875	7.068327307
	z-alpha(Bangia)	z-cornish(S)	z-cornish (R)	Price Risk (%)	LVaR (Bangia) %	LVaR (Modified) %
Mean	0.99109	3.820876166	2.948796556	2.807252427	2.885473771	3.605684819
Median	0.91061	3.972776354	2.371200501	2.602336958	2.673507938	3.374473219
Std Dev.	0.36344	0.731215626	0.942148285	1.084436172	1.0905948	1.566669063

Table 10. Portfolio risk estimates – Midcap (weights: Apollo Hospitals = .25, DLF = .25, Jindal Steel = .25 & SUN TV = .25)

	Relative Spread	Return	Skewness(S)	Kurtosis(S)	Skewness (R)	Kurtosis (R)
Mean	0.146051035	-7.761E-06	1.489623316	3.54335582	-0.140138502	1.39654264
Median	0.129639101	0.00036725	1.410208357	3.253552569	-0.134623322	1.52654544
Std Dev.	0.082192034	0.015546717	0.290370656	1.609695564	0.247151212	0.640799716

	z-alpha(Bangia)	z-cornish(S)	z-cornish (R)	Price Risk (%)	LVaR (Bangia) %	LVaR (Modified) %
Mean	0.997695483	3.382423285	2.519277212	3.350106665	3.504348381	3.958999614
Median	0.937836913	3.362695885	2.524618869	3.08827959	3.247171337	3.615072742
Std Dev.	0.357524515	0.251430845	0.207740312	1.137948206	1.149147036	1.400492434

Table 11. Portfolio risk estimates – Smallcap (weights: Bombay Dyeing = .25, Escorts = .25, Chambal Fertilizers = .25 & Gujarat Fluorochemicals = .25)

	Relative Spread	Return	Skewness(S)	Kurtosis(S)	Skewness (R)	Kurtosis (R)
Mean	0.268267967	0.000657802	1.48803039	3.781139361	-0.185756732	1.713362573
Median	0.221000049	0.000629973	1.611484413	4.326471495	-0.213558163	1.432035791
Std Dev.	0.183821156	0.020233165	0.421688997	2.172755092	0.264387721	1.006434075

	z-alpha(Bangia)	z-cornish(S)	z-cornish (R)	Price Risk (%)	LVaR (Bangia) %	LVaR (Modified) %
Mean	1.11368239	3.403449557	2.550865548	4.322830752	4.629891459	5.289657154
Median	1.030217681	3.511846396	2.582819486	4.049927902	4.370264702	4.99341017
Std Dev.	0.393071007	0.426229472	0.213752974	1.492421893	1.507556678	1.753073502

6. Backtesting Results

Using the close price as the liquidation price of the stocks instead of the mid-value of the bid and ask prices, the return values are calculated as follows:

$$Return_t = \ln \left(\frac{p_t^{close}}{p_{t-1}^{close}} \right) \quad (\text{Equation 11})$$

The value of exceedance E is taken as one if the value of the realized loss (computed using Equation 11) is larger than the predicted loss.

$$E = Return_t < -LVaR_t \quad (\text{Equation 12})$$

Table 1 contains the exact composition of the equally-weighted portfolios from diverse segments for backtesting analysis. The values of Price Risk or VaR, LVaR (Bangia) and LVaR (Modified) for the portfolios are shown in Tables 7, 8, 9, 10 and 11. The required close price of the stocks is obtained from the database Datastream for the time period from January 2010 to December 2014. Table 12 shows the magnitude of exceedances E at portfolio level for VaR, LVaR (Bangia) and LVaR (Modified) in period from January 2010 to December 2014 (Number of days = 1212). According to Kupiec's 'proportion of failures' (PF) coverage tests, only LVaR (Modified) measure is not rejected at .01 significance level.

Table 12. Magnitude of exceedances in period from January 2010 to December 2014 (1212 days)

Measure\Portfolio	Nifty	Infra	Service	Midcap	Smallcap
VaR	18	24	23	26	27
LVaR (Bangia)	12	20	21	23	21
LVaR (Modified)	9	20	13	13	10

7. Conclusion

This paper discusses the approach for calculating a portfolio-level LVaR measure by using the adapted model based on the Cornish-Fisher expansion technique used for correcting the percentiles of a standard normal distribution for non-normality. The data on Indian stocks is used for the empirical part of the analysis as research on liquidity that primarily focuses on emerging markets yield particularly powerful tests and useful independent evidence since the liquidity premium is an important feature of these data (Bekaert et al. 2007). Indian stocks belonging to diverse sectors are selected based on data availability during the period from January 2010 to December 2014. The empirical evidence shows that the portfolio LVaR (Modified) measure provides the highest risk estimates. The backtesting results demonstrate the superiority of the LVaR (Modified) estimates when compared to alternative estimation techniques. Overall, the results prove that neglecting liquidity risk or assuming that the returns are normally distributed leads to a severe underestimation of the total risk. Furthermore, the Cornish-Fisher procedure used gains accuracy with the length of the estimation horizon hence future research could address this limitation.

References

- Acharya, Viral, & Lasse Pedersen. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics*, 77(2), 375-410. <http://dx.doi.org/10.1016/j.jfineco.2004.06.007>
- Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5(1), 31-56. [http://dx.doi.org/10.1016/S1386-4181\(01\)00024-6](http://dx.doi.org/10.1016/S1386-4181(01)00024-6)
- Amihud, Y., & H. Mendelson. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2), 223-249. [http://dx.doi.org/10.1016/0304-405X\(86\)90065-6](http://dx.doi.org/10.1016/0304-405X(86)90065-6)
- Angelidis, T., & A. Benos. (2006). Liquidity adjusted value-at-risk based on the components of the bid-ask spread. *Applied Financial Economics*, 16(11), 835-851. <http://dx.doi.org/10.1080/09603100500426440>
- Bangia, A., Francis X. Diebold, Til Schuermann, & John D. Stroughair. (2002). Modeling Liquidity Risk with Implications for Traditional Market Risk Measurement and Management. *The New York University Salomon Center Series on Financial Markets and Institutions*, 8, 3-13. http://dx.doi.org/10.1007/978-1-4615-0791-8_1
- Bekaert, G., C.R Harvey, & C. Lundblad. (2007). Liquidity and Expected Returns: Lessons from Emerging Markets.

- Review of Financial Studies*, 20(6), 1783-1831. <http://dx.doi.org/10.1093/rfs/hhm030>
- Black, Fischer, & Myron Scholes. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, 81(3), 637-654. <http://dx.doi.org/10.1086/260062>
- Bongaerts, Dion, Frank De Jong, & Joost Driessen, (2011). Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market. *Journal of Finance*, 66(1), 203-240. <http://dx.doi.org/10.1111/j.1540-6261.2010.01630.x>
- Breeden, Douglas T., & Robert H. Litzenberger. (1978). Prices of State-Contingent Claims Implicit in Option Prices. *Journal of Business*, 51(4), 621-651. <http://dx.doi.org/10.1086/296025>
- Brunnermeier, Markus K., & Lasse Heje Pedersen. (2009). Market Liquidity and Funding Liquidity. *Review of Financial Studies*, 22(6), 2201-2238. <http://dx.doi.org/10.1093/rfs/hhn098>
- Chordia, T., R. Roll, & A. Subrahmanyam. (2001). Market liquidity and trading activity. *Journal of Finance*, 56(2), 501-530. <http://dx.doi.org/10.1111/0022-1082.00335>
- Ernst, C., Sebastian Stange, & Christoph Kaserer. (2012). Accounting for nonnormality in liquidity risk. *The Journal of Risk*, 14(3), 3-21. <http://dx.doi.org/10.2139/ssrn.1316769>
- Jarrow, R., & A. Subramanian, A. (1997). Mopping up liquidity. *Risk*, 170-173.
- Korajczyk, Robert, & Ronnie Sadka. (2008). Pricing the Commonality Across Alternative Measures of Liquidity. *Journal of Financial Economics*, 87(1), 45-72. <http://dx.doi.org/10.1016/j.jfineco.2006.12.003>
- Madhavan, A., M. Richardson, & M. Roomans. (1997). Why Do Security Prices Change? A Transaction-Level Analysis of NYSE Stocks. *The Review of Financial Studies*, 10(4), 1035-1064. <http://dx.doi.org/10.1093/rfs/10.4.1035>
- Pastor, Lubos, & Robert F. Stambaugh. (2003). Liquidity Risk and Expect Stock Returns. *Journal of Political Economy*, 111, 642-685. <http://dx.doi.org/10.1086/374184>
- Stange, S., & C. Kaserer. (2008). The impact of order size on stock liquidity – A representative study. *CEFS Working Paper No. 2008-9*. <http://dx.doi.org/10.2139/ssrn.1292304>
- Stange, S., & C. Kaserer. (2011). The impact of liquidity risk: a fresh look. *International Review of Finance*, 11(3), 269-301. <http://dx.doi.org/10.1111/j.1468-2443.2010.01113.x>

Notes

Note 1. BNP Paribas terminated withdrawals from three hedge funds citing “a complete evaporation of liquidity” on August 9th, 2007.

Note 2. JP Morgan, 1996, RiskMetrics – Technical Document, Fourth Edition, New York.