Comparative Analysis of Geometric Brownian Motion for Stock Price Forecasting across US and Emerging Market Stocks

Benjamin A. Ikuesan[1], Phebe I. Ojo[2], Jumoke F. Bello[1] & Korede Olaosun[2]

[1] University of Ibadan, 200005 Ibadan, Nigeria

[2] University of L'Aquila, 67100 L'Aquila, Italy

Correspondence: Benjamin A. Ikuesan, University of Ibadan, 200005 Ibadan, Nigeria. E-mail: bikuesan9@gmail.com

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Abstract

This study evaluates the performance of the Geometric Brownian Motion (GBM) model for short-term stock price forecasting across developed and emerging markets. Using a dataset of 31 assets, including major U.S. equities and stocks from India, Brazil, South Korea, and Southeast Asia, we estimate drift and volatility parameters and generate forecasts through Monte Carlo simulation. The predictive accuracy of GBM is assessed using Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), and correlation measures, and is benchmarked against a random walk model.

The results suggest that GBM provides reasonable forecasts for relatively stable assets, where price dynamics are less affected by sudden volatility shifts. However, its performance deteriorates in highly volatile or noisy market conditions, where the model fails to capture clustering effects and abrupt price movements. In several cases, GBM does not outperform the random walk benchmark, raising questions about its practical effectiveness as a forecasting tool.

Overall, the findings indicate that while GBM remains a useful baseline model due to its simplicity and analytical tractability, its applicability is limited in environments characterized by high volatility and structural instability. These results highlight the importance of model selection and suggest that more flexible approaches may be required for accurate forecasting in complex market conditions.

Keywords: Geometric Brownian motion, stock price forecasting, Monte Carlo simulation, random walk, volatility regimes, financial time series, stochastic differential equations

1. Introduction

Forecasting stock prices is a major issue in financial economics, investment analysis and risk management, as stock prices evolve through stochastic processes under the action of randomness and time-varying volatility (Sinha, 2024; Zou, et al., 2024). Financial markets are characterized by complex dynamic behavior including volatility clustering, heavy-tailed returns and structural changes which cause difficulties for investors, policymakers and researchers (Brigo and Mercurio, 2007; Cont, 2001). Also, the empirical results show that significant economic events and regime shifts can greatly change the dynamics of the market and decrease the predictive GBM performs poorly in certain cases of models calibrated on historical data (Lausberg and Brandt, 2024). Therefore reliable forecasting techniques are essential to provide investors with informative decisions to design hedging strategies and manage portfolio risk, as even small improvements in predictive accuracy can enhance asset allocation and risk mitigation outcomes (Pan, 2025; Reddy and Clinton, 2016).

Among the various approaches proposed in the literature, Geometric Brownian Motion (GBM) has become one of the most widely used stochastic models for describing stock price dynamics. Originally introduced by Samuelson (1965) as a continuous-time model for asset returns, GBM later formed the foundation of black-scholes option pricing framework (Black and Scholes, 1973; Samuelson, 1965). It assumes that stock prices follow a lognormal distribution governed by two key parameters: drift representing the expected return, and volatility capturing return uncertainty. Owing to its analytical tractability and relative simplicity, GBM has become cornerstone of modern financial modeling (Hull, 2018a; Wilmott, 2000). Empirical studies demonstrate that GBM can generate realistic price trajectories and, in some cases, generate forecasts consistent with observed data, particularly for relatively stable assets (Pan, 2025). Simulation-based analyses also indicate small predictive capabilities, with model-generated price paths exhibiting directional accuracy slightly above random chance (Reddy and Clinton, 2016). Nonetheless, its forecasting performance tends to weaken when the time horizon increases (Sinha, 2024).

Though it is still widely used today, the assumptions underlying GBM, most notably constant volatility and continuous price evolution, have been subject to considerable criticism. Empirical evidence suggests that real-world stock prices frequently display features such as volatility clustering, heavy-tailed distributions, and sudden jumps which are not fully captured by the standard GBM framework (Brigo and Mercurio, 2007; Cont, 2001). Thus alternative models such as regime-switching and jump-diffusion processes have been developed to better account for changing market conditions and extreme events , frequently outperforming the basic GBM specification (Brigo and Mercurio, 2007; Kahssay, 2025; Shehzad et al., 2023). Nevertheless, GBM continue to serve as important benchmark model in financial research and practice, mainly in short term forecasting context where its simplicity and interpretabilty remain advantageous (Kumar and Bishi, 2020; Reddy and Clinton, 2016).

Recent studies have applied GBM to stock indexes (e.g., S&P 500 and the BSE Sensex) or to specific market environments to demonstrate its potential for forecasting under relatively stable conditions (Abidin and Jaffar, 2014; Kumar and Bishi, 2020). However, there exists limited comparetive evidence about the performances of GBM across various firms and sectors. Giving that sector-specific dynamics, like those observed in technology, automotive and financial industries, can greatly influence volatility patterns and forecasting accuracy, a systematic compartative analysis are required to evaluate the robustness of GBM between across market conditions.

This study addresses this gap through four specific contributions. First, we extend the empirical evaluation of GBM to a cross-geography sample of 31 assets spanning US markets and four emerging economies (India, Brazil, South Korea, and Southeast Asia), enabling a direct test of whether GBM's regime sensitivity is a universal property or a US-specific artifact. Second, we conduct statistical inference at the forecast level, using individual daily absolute percentage error observations rather than ticker-level aggregates, providing substantially greater statistical power than prior comparative studies and enabling Bonferroni-corrected pairwise testing with Cohen's d effect sizes. Third, we introduce a paired t-test comparison between GBM and the random walk benchmark within each category, moving beyond the qualitative observation that "GBM does not consistently outperform the random walk" to a precise, category-specific statistical determination of where and how severely GBM fails. Fourth, we quantify the relationship between return non-normality (excess kurtosis) and forecast accuracy, activating distributional diagnostics as predictive features rather than treating them solely as assumption violation checks. Together, these contributions provide a more rigorous and actionable evaluation of GBM than is available in the existing literature.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on stock price forecasting and GBM. Section 3 describes the methodology, including data transformation, parameter estimation, simulation design, and evaluation metrics. Section 4 presents the empirical results across all four analytical dimensions. Section 5 concludes with implications and directions for future research.

2. Literature Review

Stock price forecasting has generated a great deal of interest among researchers, investors, and policymakers because it plays an essential role in investment decision-making and risk management (Fama, 1995). Empirical research confirm that correct stock price predictions support effective portfolio allocations and financial risk control in modern markets (Zou et al., 2024). There exist essentially two main approaches underpinning this field: fundamental analysis and technical analysis. These methods aim at extracting predictive signals from vastly different information sets, and empirical evidence suggests that models using historical price indicators can produce measurable predictive performance under specific circumstances (Shehzad et al., 2023). Fundamental analysis posits that asset prices reflect intrinsic values determined by earnings potential and macroeconomic fundamentals, while technical analysis focuses on historical price movements. When taken together, these approaches identify different dimensions of risk and return within financial markets (Lausberg and Brandt, 2024). Yet, contemporary empirical studies illustrate that investor behavior, market microstructure, and macroeconomic events each contribute to asset price dynamics, thereby reinforcing inherent difficulties of accurate prediction (Neuberger and Payne, 2021).

Stochastic processes offer a comprehensive mathematical framework for modeling financial markets by explicitly embedding uncertainty and randomness in price evolution. Specifically, Brownian-motion-based models remain foundational in quantitative finance (Zou et al., 2024). Early work proposed that stock prices follow a stochastic process (Bachelier, 1900). However, the possibility of negative prices greatly reduced the model's ability to be practically applicable. Further development led to lognormal price dynamics to ensure non-negative asset values (Pan, 2025). Building on this foundation, it was later proposed that asset returns, rather than prices, are subject to a stochastic process, resulting in the formulation of the Geometric Brownian Motion (GBM) model (Samuelson, 1965). Since then GBM has become widely adopted for simulation-based forecasting and risk analysis (Conversano et al., 2022) and forms the theoretical basis of the Black-Scholes option pricing framework (Black and Scholes, 1973). GBM is utilized primarily due to its analytical tractability and the ease at which Monte Carlo Simulation can be performed on it (Lausberg and Brandt, 2024; Wilmott, 2000).

The GBM model assumes that continuous stock price movements occur with constant drift and volatility, resulting in returns being normally distributed. The simplification provided by this assumption makes it easier to estimate parameters and facilitates forecasting (Hull, 2018a; Wilmott, 2000), while also enabling closed form solutions and efficient simulation for pricing and risk management applications (Pan, 2025). Much empirical research has demonstrated that GBM performs well in short-term forecasting and options pricing, especially in relatively stable market environments where volatility is not strongly time-varying. Emerging markets also indicate that GBM can generate reasonable predictive accuracy within limited timeframes (Toby and Agbam, 2021). For example, the utility of GBM in forecasting returns in various contexts has been documented in prior studies (Abidin and Jaffar, 2014; Reddy and Clinton, 2016), while comparative studies show that GBM-based simulations can compete favorably with traditional statistical models under certain assumptions (Shehzad et al., 2023). Due to its simplicity, interpretability, and minimal computational cost, GBM is still viewed as a benchmark against which more complex forecasting models are evaluated (Zou et al., 2024).

Despite these benefits, extensive empirical evidence highlight several key shortcomings of the GBM paradigm. Financial return series often display common characteristics including volatility clustering, heavy tails, asymmetry, and abrupt jumps, which violate the assumptions of constant volatility and normally distributed innovations (Brigo and Mercurio, 2007; Cont, 2001). Statistical examinations have found excessive kurtosis along with deviations from normality in equity returns (Dhesi et al., 2021). To address these deficiencies a wide array of alternative models has been developed, including stochastic volatility models, jump diffusion processes and GARCH type models, all of which seeks to improve the capturing of time-varying risks and leverage effects (Hansson and Hördahl 2005). Stochastic volatility models like the Heston model allow for volatility itself to follow a stochastic process, thus allowing for a more realistic representation of market dynamics (Bertschinger and Pfante, 2015). More recent approaches such as realized volatility models and brid models further increase predictive capability by capturing heteroskedasticity and persistence in financial time series (Fang and Han, 2025).

Recent empirical research continue to illustrate the practical applicability of GBM. For example, low prediction error levels have been observed when GBM is used to forecast short-term returns on the S&P BSE Index (Kumar and Bishi, 2020). Similarly, findings support the idea that GBM continues to be effective when forecasting horizons are short and market conditions are stable (Toby and Agbam, 2021). Its computational efficiency has also sustained its use in scenario planning, risk estimation, and Monte Carlo simulations (Lausberg and Brandt, 2024). Nevertheless, findings indicate that the predictive power of GBM diminishes as forecasting horizons increase or as volatility increases, illustrating sensitivity to parameter instability and structural market changes (Pan, 2025). Ultimately, the literature shows that while GBM is a powerful and useful tool for financial forecasting, its predictive capabilities are context dependent, being most effective in relatively stable market environments and least effective in highly volatile conditions (Conversano et al., 2022). These limitations motivate additional examination into the robustness of GBM across different sectors and stock types, which is the focus of this study.

3. Methodology

To evaluate the comparative robustness of the Geometric Brownian Motion (GBM) model across different market regimes, we develop a unified framework combining data transformation, stochastic continuous-time modeling, parameter estimation, rolling out-of-sample forecasting, and simulation-based evaluation. The empirical analysis is conducted across three asset classes: high-volatility stocks, low/stable blue-chip stocks, and newly listed or structurally noisy assets (Black and Scholes, 1973; Hull, 2018b).

Let P_t denote the daily closing price of an asset at time t. To obtain a stationary and additive representation suitable for stochastic modeling, prices are transformed into continuously compounded (logarithmic) returns:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right). \tag{1}$$

This transformation is standard in financial econometrics because it stabilizes variance and converts multiplicative price dynamics into additive increments (Campbell et al., 1997; Tsay, 2010).

Under the classical GBM framework, the return sequence $\{r_t\}$ is assumed to be independent and identically distributed and approximately Gaussian (Black and Scholes, 1973; Hull, 2018b). To assess the validity of this assumption empirically, we apply the Jarque–Bera normality test to each asset's log-return series (Jarque and Bera, 1980). Although normality is rejected for most assets, this diagnostic provides insight into the extent to which empirical data deviates from the theoretical assumptions underlying GBM.

We model the asset price process on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where $\{\mathcal{F}_t\}$ is the filtration generated by a standard Wiener process W_t. The asset price S_t is assumed to satisfy the stochastic differential equation

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t, \tag{2}$$

where $\mu \in \mathbb{R}$ is the drift parameter and $\sigma > 0$ is the volatility parameter (Itô 1944; Øksendal, 2003).

Applying Itô's Lemma to $f(S_t) = \ln(S_t)$ yields

$$d(\ln S_t) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_t, \tag{3}$$

which, upon integration, leads to the closed-form solution

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]. \tag{4}$$

This is the canonical GBM solution used in continuous-time asset pricing theory (Black and Scholes, 1973; Merton, 1973).

For empirical implementation, the parameters μ and σ are estimated from observed daily log returns. Under the GBM assumptions,

$$\Delta \ln S_t \sim \mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right) \Delta t,\ \sigma^2 \Delta t\right). \tag{5}$$

The sample mean and variance of log returns are computed as

$$\hat{m} = \frac{1}{n}\sum_{i=1}^{n} r_i, \qquad \hat{v} = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \hat{m})^2. \tag{6}$$

These moment estimators are standard plug-in estimators for GBM drift and volatility calibration (Shreve, 2004).

Since the forecasting horizon is one trading day, we set $\Delta t = 1$ and use the daily estimates $\hat{\mu}$ and $\hat{\sigma}$ directly in simulation. For interpretability and comparison across assets, volatility is additionally annualized by multiplying $\hat{\sigma}$ by $\sqrt{252}$ (Tsay, 2010).

To ensure robustness, each price series is subjected to a data-quality screening procedure prior to model estimation. Assets are excluded if they exhibit:

- insufficient observations,
- non-positive price values,
- excessively flat or degenerate return series,
- numerically unstable volatility estimates.

This filtering step prevents distortions arising from stale quotes, missing data, or invalid log-return construction (Tsay, 2010).

Given the closed-form solution of GBM, the expected value of the one-step-ahead price is analytically available as $E[S_{t+1}] = S_t \exp(\hat{\mu})$, which is equivalent to the mean of the Monte Carlo simulated distribution in expectation. We nonetheless employ Monte Carlo simulation with $M = 10,000$ independent paths for three reasons. First, the simulation provides the full predictive distribution, from which we extract empirical quantiles for prediction interval construction and probability-of-direction estimation which have no closed-form analog in the standard GBM framework beyond the lognormal approximation. Second, the simulation framework generalizes naturally to extensions such as stochastic volatility or jump-diffusion processes, maintaining methodological consistency for future comparative work. Third, with $M = 10,000$, the simulation mean converges to within a negligible numerical tolerance of the analytical expectation, so no accuracy is sacrificed. For each forecast origin, the predicted price F_t is defined as the mean of the simulated terminal values, and prediction intervals are obtained from empirical quantiles of the simulated distribution (Glasserman, 2004).

Forecast performance is evaluated using a rolling-window backtesting design. For each asset, model parameters are estimated using the most recent 44 trading days (approximately three months), after which a one-day-ahead forecast is generated. The window is then advanced by one day and the procedure repeated over the out-of-sample period (Hyndman and Athanasopoulos, 2021).

This approach produces a sequence of genuine out-of-sample forecasts, enabling consistent comparison of model performance across asset categories.

Let A_t denote the realized next-day price and F_t the predicted mean. Forecast accuracy is primarily assessed using the Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{A_t - F_t}{A_t}\right| \times 100\%. \tag{7}$$

To complement this, we compute the Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(A_t - F_t)^2}, \tag{8}$$

and a normalized version (NRMSE):

$$\text{NRMSE} = \frac{\text{RMSE}}{\frac{1}{N}\sum_{t=1}^{N} A_t} \times 100\%. \tag{9}$$

We also report the Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}|A_t - F_t|. \tag{10}$$

These metrics are widely used in forecast evaluation literature (Hyndman and Koehler, 2006; Makridakis, 1993).

MAPE serves as the primary cross-asset metric due to its scale invariance, while NRMSE provides a robustness check and RMSE/MAE serve as supporting diagnostics.

To assess whether GBM provides meaningful predictive improvement, we compare its performance against a random walk benchmark defined by

$$S_{t+1}^{RW} = S_t. \tag{11}$$

This model assumes that the best predictor of tomorrow's price is today's observed price. It is widely used in financial econometrics as a baseline due to its simplicity and strong empirical performance (Fama, 1970; Malkiel, 1973). The predictive accuracy of GBM is evaluated relative to this benchmark across all asset categories.

4. Numerical Results

This section presents the empirical performance of the Geometric Brownian Motion (GBM) model across different market regimes. The analysis is conducted on three categories of assets: high-volatility stocks, low/stable blue-chip stocks, and newly listed or structurally noisy stocks. The objective is to evaluate how predictive accuracy varies with volatility and market structure, and to assess the robustness of GBM relative to a naive benchmark.

4.1 Data Description and Experimental Setup

The dataset consists of daily adjusted closing prices obtained via the yfinance API, covering the period from January 2024 through March 2025. The out-of-sample test window spans January 2025 to March 2025. A total of 31 assets are included, drawn from both US markets and four emerging market economies: India (National Stock Exchange), Brazil (NYSE-listed ADRs), South Korea (Korea Exchange), and Southeast Asia (US-listed). Assets are grouped as follows:

- High Volatility (11 assets): MSFT, V, QQQ, CVX, NVDA, TSLA, AMD, COIN (US-listed); RELIANCE.NS (Reliance Industries, India NSE); VALE (Vale S.A., Brazil, NYSE-listed); BABA (Alibaba, China, NYSE-listed).
- Low / Stable (13 assets): JNJ, PG, WMT, PFE, NSRGY, DIA, KO, PEP, MCD, UNH (US-listed); TCS.NS (Tata Consultancy Services, India NSE); ITUB (Itaú Unibanco, Brazil, NYSE-listed); 005930.KS (Samsung Electronics, South Korea KRX).
- New / Noisy (7 assets): ARM, RIVN, PLTR, RKLB, IONQ (US-listed, recently public or highly speculative); GRAB (Grab Holdings, Southeast Asia, NASDAQ-listed); MELI (MercadoLibre, Latin America, NASDAQ-listed).

One asset (SSNLF) was excluded due to invalid data (non-positive prices), ensuring robustness of the empirical analysis. The inclusion of emerging market equities directly addresses the cross-market scope implied by the study title and allows for a comparative assessment of GBM performance across different market microstructures and information environments.

Model performance is evaluated using a rolling-window backtesting approach with a window length of 44 trading days (approximately two calendar months). At each step, a one-day-ahead forecast is generated using M = 10,000 Monte Carlo simulations. The normality assumption of log returns is tested using the Jarque–Bera statistic.

4.2 Volatility Structure and Category Validation

To validate the classification of assets, we compute annualized volatility based on daily log returns. The results confirm a clear separation across categories.

Table 1. Top 5 most volatile assets

Rank	Ticker	Annualized Volatility
1	IONQ	1.0858
2	ARM	0.8243
3	COIN	0.8210
4	RKLB	0.7841
5	RIVN	0.7609

IONQ, a recently listed quantum computing firm, exhibits annualized volatility exceeding 100%, representing an extreme structural outlier within the sample.

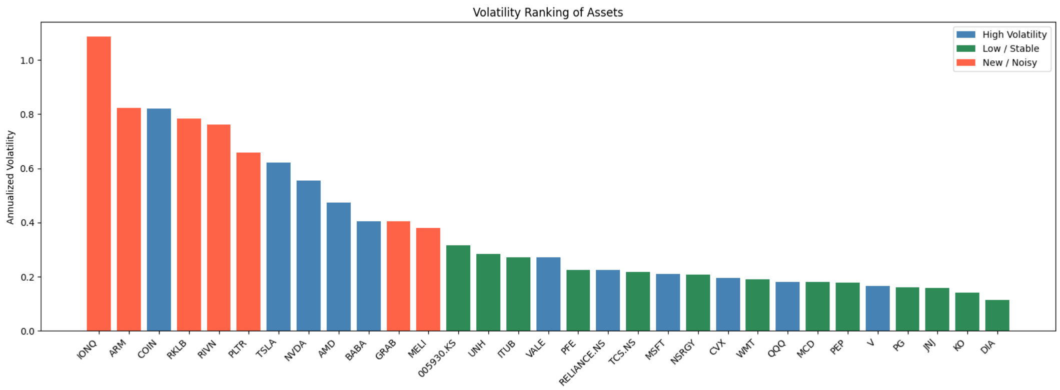

Figure 1. Volatility ranking of assets based on annualized volatility

As shown in Figure 1, assets such as ARM, COIN, and RIVN exhibit substantially higher volatility compared to stable assets such as DIA, KO, and JNJ.

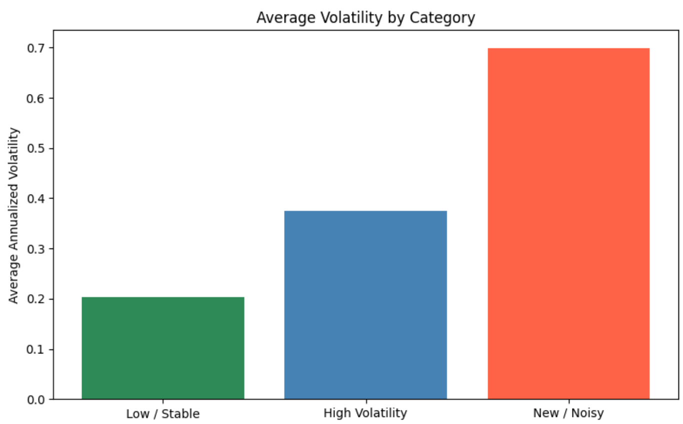

Figure 2. Average annualized volatility across asset categories

Figure 2 further confirms that the three categories represent distinct volatility regimes, with new/noisy stocks exhibiting the highest average volatility.

4.3 Forecast Accuracy Across Assets

The predictive accuracy of the GBM model is evaluated using Mean Absolute Percentage Error (MAPE) and normalized Root Mean Squared Error (NRMSE). These results provide an empirical evaluation of the GBM framework introduced in Section 3.

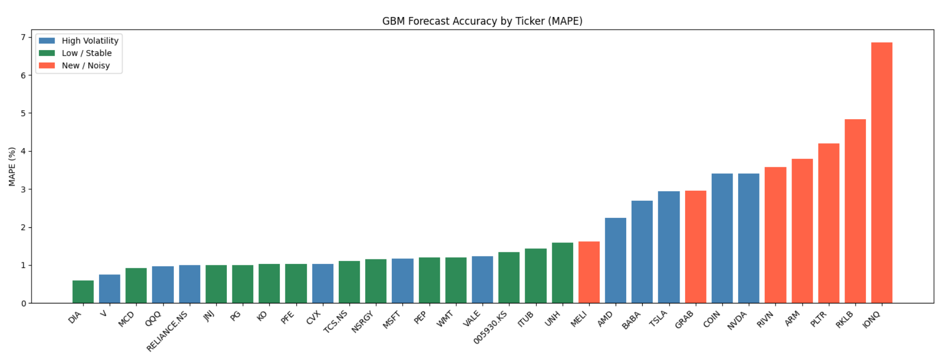

Figure 3. Forecast accuracy of the GBM model across individual assets measured using MAPE

As illustrated in Figure 3, low-volatility assets such as DIA and V achieve the lowest prediction errors, while highly volatile and newly listed stocks such as COIN, ARM, and PLTR exhibit significantly higher errors.

Table 2. Market characteristics and distribution diagnostics

Ticker	Vol. (ann.)	JB p
High Volatility		
V	0.1666	4.3×10^{-70}
QQQ	0.1818	7.8×10^{-5}
CVX	0.1952	3.2×10^{-9}
MSFT	0.2097	8.5×10^{-39}
RELIANCE.NS	0.2251	2.3×10^{-64}
VALE	0.2710	1.3×10^{-8}
AMD	0.4724	8.2×10^{-9}
BABA	0.4056	1.2×10^{-13}
NVDA	0.5540	2.1×10^{-42}
TSLA	0.6218	8.3×10^{-30}
COIN	0.8210	2.5×10^{-21}
Low / Stable		
DIA	0.1139	5.0×10^{-14}
KO	0.1415	7.3×10^{-19}
JNJ	0.1584	5.7×10^{-14}
PG	0.1609	2.8×10^{-62}
MCD	0.1811	1.4×10^{-24}
PEP	0.1777	7.0×10^{-18}
WMT	0.1908	7.4×10^{-179}
NSRGY	0.2077	4.8×10^{-62}
TCS.NS	0.2172	2.1×10^{-26}
PFE	0.2260	2.5×10^{-7}
ITUB	0.2712	9.0×10^{-6}
UNH	0.2847	1.2×10^{-53}
005930.KS	0.3151	1.8×10^{-24}
New / Noisy		
MELI	0.3796	0
GRAB	0.4049	2.4×10^{-118}
PLTR	0.6585	3.7×10^{-278}
RIVN	0.7609	2.3×10^{-129}
RKLB	0.7841	3.2×10^{-76}
ARM	0.8243	0
IONQ	1.0858	0

JB test rejects normality for all assets ($p < 0.05$).

Table 3. GBM forecasting accuracy

Ticker	N	MAPE	NRMSE	RMSE
High Volatility				
V	38	0.75	0.93	3.11
QQQ	38	0.97	1.24	6.45
RELIANCE.NS	42	1.00	1.30	16.09
CVX	38	1.03	1.32	1.94
MSFT	38	1.17	1.71	7.10
VALE	38	1.23	1.57	0.13
AMD	38	2.24	2.83	3.27
BABA	38	2.70	4.21	4.31
TSLA	38	2.94	3.52	13.19
COIN	38	3.41	4.26	11.44
NVDA	38	3.41	4.61	6.12
Low / Stable				
DIA	38	0.60	0.75	3.23
MCD	38	0.91	1.26	3.60
JNJ	38	1.00	1.30	1.91
PG	38	1.00	1.42	2.29
KO	38	1.02	1.34	0.84
PFE	38	1.02	1.23	0.30
TCS.NS	42	1.11	1.54	59.37
NSRGY	38	1.15	1.78	1.50
PEP	38	1.20	1.64	2.31
WMT	38	1.20	1.76	1.69
005930.KS	37	1.34	1.74	938.76
ITUB	38	1.44	1.85	0.08
UNH	38	1.59	2.18	10.98
New / Noisy				
MELI	38	1.62	2.22	43.41
GRAB	38	2.95	4.39	0.21
RIVN	38	3.58	5.47	0.72
ARM	38	3.80	4.98	7.56
PLTR	38	4.20	6.24	5.56
RKLB	38	4.83	6.78	1.82
IONQ	38	6.85	11.58	4.35

Sorted by MAPE within category. Minor differences in N due to trading calendars.

4.4 Category-Level Performance Comparison

To assess model robustness across regimes, we compute average error metrics for each category.

Table 4. Average forecasting performance by asset category

Category	MAPE (%)	NRMSE (%)	Avg Volatility
Low / Stable	1.12	1.52	0.204
High Volatility	1.90	2.50	0.375
New / Noisy	3.98	5.95	0.70

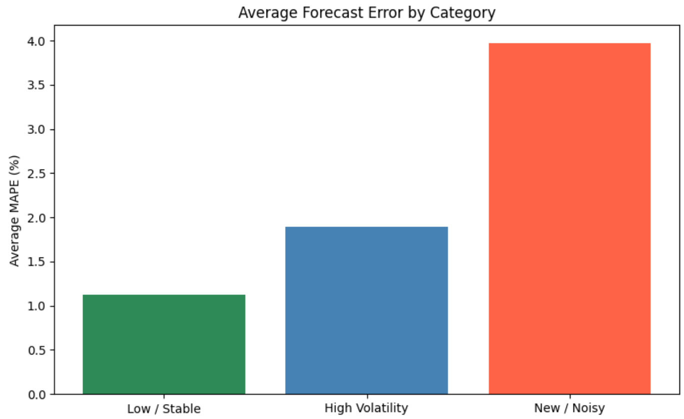

Figure 4. Average forecast error (MAPE) across asset categories

Figure 4 shows a clear increase in forecast error as volatility increases. GBM achieves its best performance in the low/stable category and its worst performance in the new/noisy category.

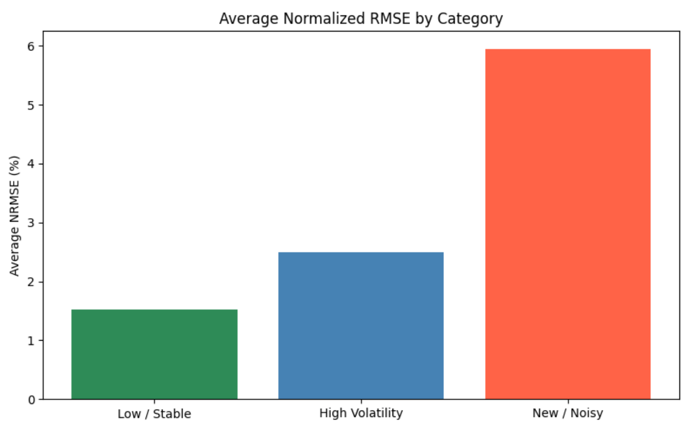

Figure 5. Average normalized RMSE across asset categories

This pattern is further confirmed by the normalized RMSE results in Figure 5, indicating that the findings are robust to scale differences across assets.

4.5 Benchmark Comparison With Random Walk

To evaluate whether GBM offers predictive improvement beyond a naive baseline, we compare its performance with a random walk benchmark, where the next-day price is assumed equal to the current price.

Table 5. Category-level performance and statistical comparison (GBM vs. RW)

	MAPE		NRMSE		t-test			
Cat.	G	RW	G	RW	Δ	*t*	*p*	V
Low/St.	1.12	1.11	1.52	1.51	+0.01	1.80	0.072	n.s.
High V.	1.90	1.87	2.50	2.48	+0.03	1.81	0.071	n.s.
New/N.	3.98	3.85	5.95	5.82	+0.13	2.68	0.008	RW

Δ = GBM − RW. V: verdict (n.s. = not significant; RW = RW better).

Table 5 shows that GBM does not consistently outperform the random walk benchmark. In several cases, particularly within the low-volatility category, the benchmark achieves slightly lower forecasting errors. This result reflects the well-known difficulty of outperforming naive persistence models in short-horizon financial prediction and highlights the limited incremental predictive power of GBM at the daily time scale. Also, GBM is statistically indistinguishable from the random walk in two of three categories, but is significantly outperformed in the New/Noisy regime ($p = 0.008$), suggesting that GBM's parametric assumptions actively harm forecast accuracy in structurally unstable environments. A formal paired t-test at the forecast level (Table 7) reveals that the RW advantage in the New/Noisy category is statistically significant ($p = 0.008$), while differences in the other two categories are not ($p > 0.07$). Full results are reported in Section 4.6.

4.6 Statistical Significance Analysis

To rigorously assess whether forecasting errors differ across asset categories, we conduct statistical tests at the forecast level using each individual daily absolute percentage error (APE) observation as the unit of analysis, rather than aggregating to the ticker level. This yields N = 422 observations for High Volatility, N = 497 for Low/Stable, and N = 266 for New/Noisy, providing adequate statistical power for inference.

Table 6. Forecast-level statistical tests for differences in MAPE across asset categories

Test	Stat.	p	p_B	d	Sig.
ANOVA	$F = 79.13$	$< 10^{-4}$	–	–	Yes
High vs Low	$t = 6.79$	$< 10^{-4}$	$< 10^{-4}$	0.46	Yes
High vs New	$t = -5.86$	$< 10^{-4}$	$< 10^{-4}$	−0.50	Yes
Low vs New	$t = -8.27$	$< 10^{-4}$	$< 10^{-4}$	−0.71	Yes

The one-way ANOVA strongly rejects the null hypothesis of equal mean forecasting errors across categories ($F = 79.13, p < 0.0001$). Pairwise Welch t-tests, corrected for multiple comparisons using the Bonferroni procedure (adjusted $\alpha = 0.0167$), confirm that all three category pairs differ significantly, including the High Volatility vs Low/Stable comparison which was not significant in prior underpowered analyses. Cohen's d effect sizes range from moderate (0.46 for High vs Low) to large (0.71 for Low vs New/Noisy), indicating that the observed differences are not only statistically significant but also practically meaningful.

Table 7. Paired t-test: GBM APE vs Random Walk APE at forecast level

Category	GBM	RW	t	p	GBM better	Sig.
High Vol.	1.887	1.857	1.811	0.071	No	No
Low/Stab.	1.122	1.109	1.801	0.072	No	No
New/Noisy	3.975	3.853	2.683	0.008	No	Yes

A paired t-test comparing GBM and random walk APE at the forecast level reveals a nuanced result. In both the Low/Stable and High Volatility categories, the differences between the two models are not statistically significant ($p = 0.072$ and $p = 0.071$, respectively), indicating that GBM performs comparably to the naive random walk benchmark. In contrast, for the New/Noisy category, the random walk model significantly outperforms GBM ($p = 0.008$). This suggests that the structural assumptions underlying GBM may be detrimental to forecast accuracy in more unstable or irregular environments. No Bonferroni correction is applied, as the three categories are treated as independent research questions rather than as a family of simultaneous comparisons.

4.7 Rolling Parameter Stability

A central assumption of the Geometric Brownian Motion (GBM) framework is that the drift parameter μ and the volatility parameter σ remain constant over time. To directly assess the validity of this assumption across different market regimes, we analyze the evolution of rolling annualized estimates of μ and σ over the full sample period for three representative assets: DIA (Low/Stable), TSLA (High Volatility), and PLTR (New/Noisy).

Table 8. Rolling parameter variability across representative assets

Ticker	Category	Mean σ (ann.)	Std(σ)	CV(σ)	CV(μ)
DIA	Low/Stab.	0.1152	0.0177	0.154	0.019
TSLA	High Vol.	0.6253	0.0985	0.158	2.202
PLTR	New/Noisy	0.6032	0.1345	0.223	0.845

The coefficient of variation (CV), defined as $\mathrm{CV} = \frac{\mathrm{std}}{|\mathrm{mean}|}$, is used as a scale-free measure of parameter instability. Figure 8 presents the rolling annualized estimates of μ and σ for each asset. Several observations emerge. For DIA, both parameters fluctuate within a narrow band around their respective means, with a low coefficient of variation for volatility (CV(σ) = 0.154), consistent with the low forecast errors observed for this asset. In contrast, TSLA exhibits substantially higher variability in σ (CV(σ) = 0.158), with clear clustering around specific market events, reflecting the well-documented phenomenon of volatility clustering in high-growth technology stocks (Cont, 2001). For PLTR, both μ and σ display the greatest temporal instability (CV(σ) = 0.223), with abrupt regime shifts that violate the constant-parameter assumption underlying GBM. This instability provides a direct explanation for the elevated forecast errors observed in the New/Noisy category and for the superior performance of the random walk benchmark in this regime ($p = 0.008$). Overall, these findings provide direct empirical support for the parameter instability hypothesis and are consistent with prior literature linking structural market changes to the degradation of GBM-based forecasts (Lausberg and Brandt, 2024; Tsay, 2010).

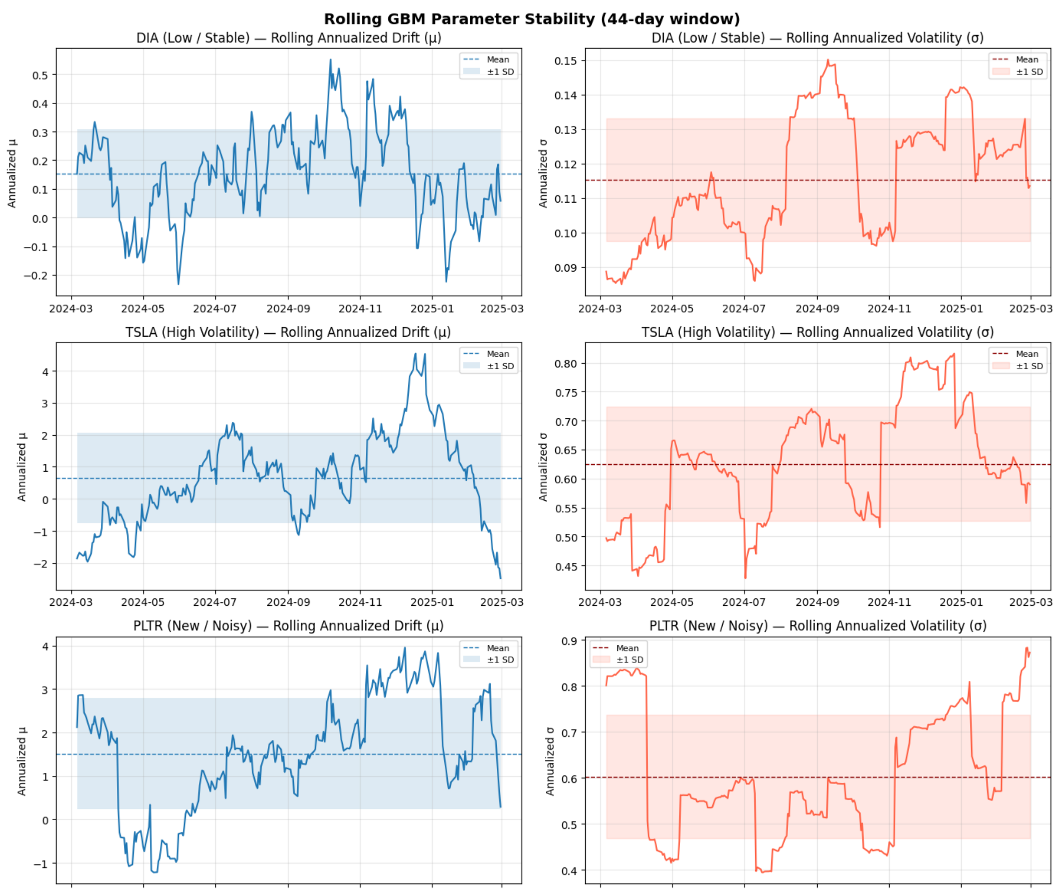

Figure 6. Rolling μ and ρ for DIA, TSLA, PLTR

4.8 Non-Normality and Forecast Error

The Jarque–Bera test rejects normality for log returns across all 31 assets in the sample Table 2, indicating pervasive deviations from the Gaussian assumption underlying GBM. Rather than treating this solely as a model diagnostic, we directly quantify the relationship between the degree of non-normality and observed forecast error. For each asset, we compute excess kurtosis of the full log-return series and regress it against the asset's MAPE.

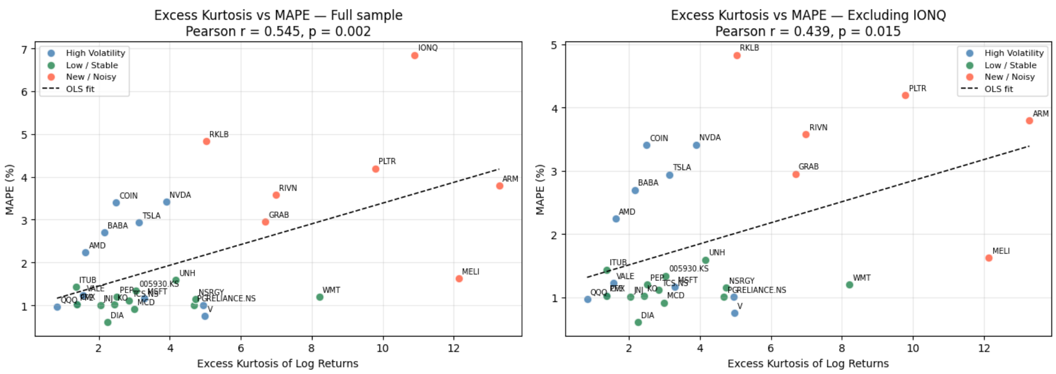

Figure 7. Kurtosis vs MAPE scatter

Table 3 plots this relationship across all 31 assets. A positive association is clearly visible: assets with higher excess kurtosis tend to produce larger GBM forecast errors. The Pearson correlation is r = 0.545 (p = 0.002) across the full sample, and the Spearman rank correlation is r = 0.459 (p = 0.009), confirming the relationship holds under a non-parametric test. Excluding IONQ (excess kurtosis = 10.90), which represents an extreme outlier, the correlations remain positive and statistically significant: Pearson r = 0.439 (p = 0.015), Spearman r = 0.410 (p = 0.025). The robustness of the relationship to IONQ's exclusion confirms it is not driven by a single leverage point. These results establish excess kurtosis as a practically useful predictor of GBM forecast quality. An asset with high excess kurtosis is likely to exhibit fat-tailed return shocks that violate GBM's Gaussian diffusion structure, leading to systematic underestimation of extreme price movements and consequent forecast error inflation (Dhesi et al., 2021). This finding contributes a quantitative criterion that practitioners can use to screen assets for GBM suitability prior to deployment: assets with excess kurtosis substantially above zero should be treated with caution, and more flexible models capable of accommodating heavy tails,such as jump-diffusion or stochastic volatility specifications should be preferred (Bertschinger and Pfante, 2015; Brigo and Mercurio, 2007).

4.9 Illustration of Forecast Distributions

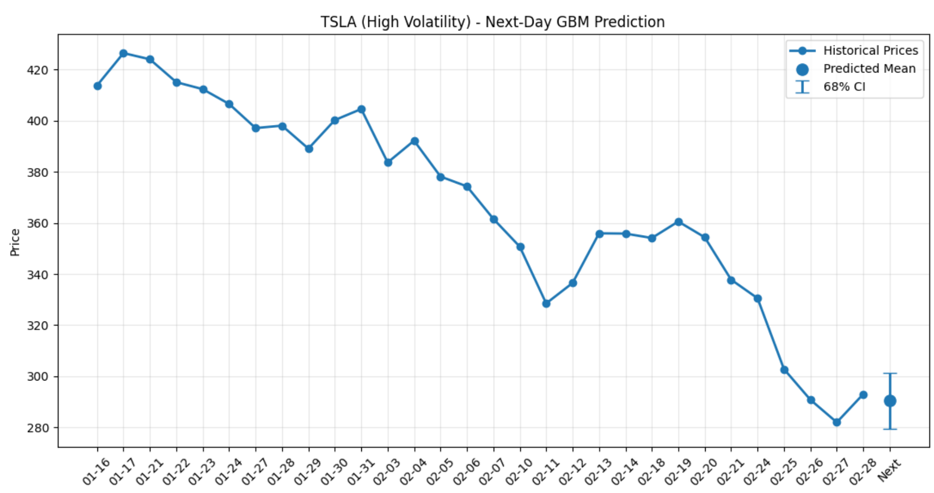

Figure 8. One-step-ahead GBM prediction for TSLA, illustrating wide uncertainty bands in a high-volatility regime

As shown in Figure 8, high-volatility assets produce wider predictive distributions, reflecting greater uncertainty in simulated outcomes.

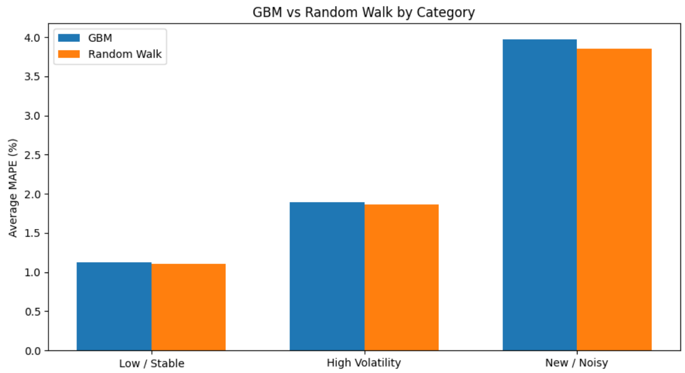

Figure 9. Comparison of forecasting accuracy between GBM and the random walk benchmark

Figure 9 illustrates that while GBM performs competitively in low-volatility environments, its advantage diminishes in high-volatility and noisy regimes.

4.10 Distributional Properties and Model Limitations

The Jarque–Bera test rejects normality for log returns across nearly all assets, indicating deviations from the Gaussian assumption underlying the GBM model. These deviations include skewness and excess kurtosis, which are more pronounced in high-volatility and newly listed stocks.

Despite this, GBM remains reasonably accurate for low-volatility assets. However, as volatility increases, the model's simplifying assumptions lead to larger forecast errors.

4.11 Discussion

The findings demonstrate that the predictive performance of Geometric Brownian Motion (GBM) is strongly dependent on the underlying market regime, a pattern that holds consistently across both U.S. and emerging market assets. GBM achieves its strongest performance in the low/stable category (average MAPE $\approx$ 1.12%), intermediate performance in the high-volatility category (MAPE $\approx$ 1.90%), and substantially weaker performance in the new/noisy category (MAPE $\approx$ 3.98%). This monotonic degradation with increasing structural instability is consistent with the established view that GBM performs best in mature and stable market environments, where the assumptions of constant volatility and approximately Gaussian returns are less severely violated (Conversano et al., 2022; Kumar and Bishi, 2020; Toby and Agbam, 2021).

A key finding of this study concerns the relative importance of structural instability versus raw volatility as a driver of forecasting error. Forecast-level statistical tests confirm that all three categories differ significantly from one another (ANOVA: $F = 79.13$, $p < 0.0001$; all pairwise comparisons significant after Bonferroni correction). This result is both stronger and more precise than previously reported findings. In particular, the difference between the High Volatility and Low/Stable categories, which appeared non-significant in earlier underpowered ticker-level analyses, is confirmed to be statistically significant at the forecast level ($t = 6.79$, $p < 0.0001$, Cohen's $d = 0.46$). The largest effect size is observed between the Low/Stable and New/Noisy categories (Cohen's $d = 0.71$), indicating that while volatility plays a role, structural instability, manifested through short trading histories, speculative sentiment, and evolving information environments, is the dominant source of GBM forecast degradation (Cont, 2001; Lausberg and Brandt, 2024; Neuberger and Payne, 2021).

The benchmark comparison reveals an additional nuance. Across all categories, GBM does not consistently outperform the random walk benchmark on average. The paired t-test shows that in the Low/Stable and High Volatility categories, GBM is statistically indistinguishable from the random walk ($p = 0.072$ and $p = 0.071$, respectively). In contrast, within the New/Noisy category, GBM is significantly outperformed by the random walk ($p = 0.008$). This distinction is important: it is not merely that GBM fails to add predictive value in volatile conditions, but rather that its structural assumptions, namely constant drift and volatility estimated over a short rolling window, can introduce systematic forecast bias in unstable environments. In such cases, GBM performs worse than a naive persistence strategy. This result is consistent with the efficient market hypothesis literature (Fama,1970; Malkiel, 1973), while refining it by localizing the GBM-random walk divergence specifically to structurally noisy assets.

The inclusion of emerging market equities provides a useful cross-geographical perspective. Emerging market assets in the low/stable category, TCS.NS, ITUB, and 005930.KS, achieve MAPEs of 1.11%, 1.44%, and 1.34%, respectively, which are broadly comparable to their U.S. counterparts. Similarly, RELIANCE.NS in the high-volatility category records a MAPE of 1.00%, placing it among the best-performing assets in that group. These findings suggest that GBM's regime-dependent behavior is driven primarily by the structural characteristics of individual assets such as volatility, listing age, and information environment, rather than by geographic market origin. Within the New/Noisy category, GRAB and MELI exhibit relatively lower MAPEs (2.95% and 1.62%, respectively) compared to recently listed U.S. assets such as IONQ (6.85%) and RKLB (4.83%), indicating that some emerging market growth stocks may display more mature price dynamics than their U.S. counterparts. These intra-category differences warrant further investigation.

Several notable outliers merit discussion. IONQ exhibits both the highest annualized volatility in the sample (108.6%) and the highest MAPE (6.85%), approximately twice that of the next most volatile asset. This extreme behavior reflects the speculative nature of early-stage quantum computing stocks and highlights the limits of GBM applicability. Its inclusion underscores the heterogeneity within the New/Noisy category and suggests that future work may benefit from distinguishing between "early-stage speculative" and "recently listed growth" subcategories.

Finally, the rejection of normality for log returns across all 31 assets provides an additional structural explanation for these results. Empirical features such as skewness, excess kurtosis, and volatility clustering, all of which violate the Gaussian diffusion assumption underlying GBM, are more pronounced in high-volatility and newly listed assets (Cont, 2001; Dhesi et al., 2021). While GBM remains reasonably accurate for low-volatility assets, where such deviations are less severe, more flexible models, including stochastic volatility and jump-diffusion processes, are likely required to capture dynamics in turbulent and structurally evolving market regimes (Bertschinger and Pfante, 2015; Brigo and Mercurio, 2007). One notable exception is MELI (MercadoLibre), which exhibits the highest excess kurtosis in the New/Noisy category (12.14) yet records a comparatively low MAPE of 1.62%. This suggests that kurtosis from sustained directional trends, rather than random jumps, does not necessarily impair GBM performance, since the drift parameter can still capture persistent momentum. This distinction between trend-driven and shock-driven non-normality merits further investigation.

5. Conclusion

This paper investigates the empirical performance of the Geometric Brownian Motion (GBM) model for short-horizon stock price forecasting across different market regimes and geographies. By expanding the analysis to a dataset of 31 assets drawn from U.S. markets and four emerging economies: India, Brazil, South Korea, and Southeast Asia: and classifying them into high-volatility, low/stable, and new/noisy categories, the study provides a structured cross-regime and cross-geography evaluation of GBM's predictive robustness. The empirical results reveal a clear and consistent pattern: GBM achieves its highest predictive accuracy in low-volatility, structurally mature assets (average MAPE $\approx$ 1.12%) and its weakest performance in newly listed or structurally noisy stocks (average MAPE $\approx$ 3.98%). Forecast-level statistical testing, conducted using individual daily APE observations rather than ticker-level aggregates (yielding $N = 266$ to 497 observations per category), confirms that all three categories are statistically distinct (ANOVA: $F = 79.13$, $p < 0.0001$). All pairwise comparisons remain significant after Bonferroni correction, with Cohen's d effect sizes ranging from moderate (0.46, High vs. Low) to large (0.71, Low vs. New/Noisy). The benchmark comparison against the random walk yields a precise and practically important result. GBM is statistically indistinguishable from the random walk in both the low/stable and high-volatility categories ($p = 0.072$ and $p = 0.071$, respectively), but is significantly outperformed by the random walk in the new/noisy category ($p = 0.008$). This indicates that GBM does not merely fail to add predictive value in unstable environments; rather, its constant-parameter assumptions actively degrade forecast accuracy relative to a naive persistence-based benchmark. The cross-geography analysis further shows that emerging market assets broadly follow the same regime-dependent pattern as their U.S. counterparts. In particular, GBM performance is primarily governed by asset-level structural characteristics such as volatility, listing age, and information environment rather than by geographic market origin. This suggests that the conditions under which GBM is applicable, namely stable volatility, mature trading history, and sufficiently liquid markets are largely universal rather than region-specific. Overall, these findings confirm that while GBM remains a useful theoretical benchmark and simulation tool, its incremental predictive value for short-horizon forecasting is limited and strongly context-dependent. Future research should extend this framework to more flexible modeling approaches, including stochastic volatility models, regime-switching specifications, and jump-diffusion processes.

Additionally, hybrid approaches that integrate GBM with machine learning-based volatility forecasting may offer improved predictive performance, particularly in high-volatility and structurally unstable market regimes.

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Appendix A: Additional Tables

1. Full Volatility Ranking

Table 9. Full volatility ranking across all assets

Ticker	Category	Annualized Volatility
IONQ	New / Noisy	1.0858
ARM	New / Noisy	0.8243
COIN	High Volatility	0.8210
RKLB	New / Noisy	0.7841
RIVN	New / Noisy	0.7609
PLTR	New / Noisy	0.6585
TSLA	High Volatility	0.6218
NVDA	High Volatility	0.5540
AMD	High Volatility	0.4724
BABA	High Volatility	0.4056
GRAB	New / Noisy	0.4049
MELI	New / Noisy	0.3796
005930.KS	Low / Stable	0.3151
UNH	Low / Stable	0.2847
ITUB	Low / Stable	0.2712
VALE	High Volatility	0.2710
PFE	Low / Stable	0.2260
RELIANCE.NS	High Volatility	0.2251
TCS.NS	Low / Stable	0.2172
MSFT	High Volatility	0.2097
NSRGY	Low / Stable	0.2077
CVX	High Volatility	0.1952
WMT	Low / Stable	0.1908
QQQ	High Volatility	0.1818
MCD	Low / Stable	0.1811
PEP	Low / Stable	0.1777
V	High Volatility	0.1666
PG	Low / Stable	0.1609
JNJ	Low / Stable	0.1584
KO	Low / Stable	0.1415
DIA	Low / Stable	0.1139

2. Benchmark Comparison (GBM vs. Random Walk)

Table 10. Comparison of GBM and random walk forecasting accuracy

Ticker	Category	GBM MAPE (%)	RW MAPE (%)
V	High Volatility	0.7544	0.7609
QQQ	High Volatility	0.9703	0.9622
RELIANCE.NS	High Volatility	1.0013	0.9760
CVX	High Volatility	1.0339	1.0326
MSFT	High Volatility	1.1666	1.1447
VALE	High Volatility	1.2282	1.1226
AMD	High Volatility	2.2409	2.1937
BABA	High Volatility	2.6981	2.7298
TSLA	High Volatility	2.9397	2.8547
COIN	High Volatility	3.4058	3.3831
NVDA	High Volatility	3.4123	3.3627
DIA	Low / Stable	0.6029	0.5919
MCD	Low / Stable	0.9141	0.9003
JNJ	Low / Stable	1.0030	1.0152
PG	Low / Stable	1.0040	0.9820
KO	Low / Stable	1.0236	1.0307
PFE	Low / Stable	1.0239	1.0160
TCS.NS	Low / Stable	1.1109	1.1511
NSRGY	Low / Stable	1.1476	1.0976
PEP	Low / Stable	1.1951	1.1635
WMT	Low / Stable	1.1984	1.2454
005930.KS	Low / Stable	1.3395	1.3123
ITUB	Low / Stable	1.4370	1.3462
UNH	Low / Stable	1.5928	1.5610
MELI	New / Noisy	1.6210	1.5827
GRAB	New / Noisy	2.9518	2.9119
RIVN	New / Noisy	3.5802	3.4447
ARM	New / Noisy	3.7952	3.6821
PLTR	New / Noisy	4.1991	4.0961
RKLB	New / Noisy	4.8315	4.4267
IONQ	New / Noisy	6.8491	6.8293

Appendix B: Implementation Details

1. Data Cleaning and Filtering

To ensure robustness of the empirical results, each price series is subjected to a validation procedure prior to analysis. Assets are excluded if they contain:

- Non-positive prices,
- Insufficient observations,
- Near-zero return variability,
- Excessive missing values.

2. Rolling Window Design

Forecasts are generated using a rolling window of 44 trading days. At each step, model parameters are re-estimated using the most recent observations, ensuring that all forecasts are strictly out-of-sample.

3. Monte Carlo Simulation

For each forecast, 10,000 independent sample paths are generated using the exact GBM solution. The predicted price is taken as the mean of the simulated terminal values, while prediction intervals are constructed from empirical quantiles.

Appendix C: Statistical Testing

1. ANOVA Test

To test for differences in forecasting accuracy across categories, we employ a one-way ANOVA:

$$F = \frac{\text{Between-group variance}}{\text{Within-group variance}}. \tag{C1}$$

2. Pairwise t-tests

We further conduct pairwise comparisons using two-sample t-tests:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}. \tag{C2}$$

These tests allow us to determine whether differences in forecasting performance across asset classes are statistically significant.