Knowledge of Slope Concept in Mathematics Textbooks in Undergraduate Education

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Abstract

This study questions the dominant approach in the presentation of the concept of slope in undergraduate education through mathematics textbooks. For this purpose, five mathematics textbooks, were analyzed for problems related to slope, and similarities and differences were revealed. First, the subjects related to the slope in the textbooks were categorized and examined within the context in which they were handled. Therefore, this study is a qualitative study that adopts the interpretive paradigm. These categories are discussed in the form of connectivity, exploration, and purpose using the study for the context (Rezat, 2006). Stump (1999; 2001b) and Moore-Russo et al. (2011) studies were used for cognitive development. These are determined as geometric ratio, behavioral indicator, property determiner, algebraic ratio, parametric coefficient, functional property, linear constant, real life, physical property and trigonometry. Representations of the process skills were chosen as algebraic expressions, tables, and graphics. The concept is discussed in the form of calculus. The analysis in the current study also took into account the use of technology (Akkoyunlu, 2002; Schware & Jaramillo, 1998) CAS and Scientific and Graphing Calculators. Definition, justification, and explanation for performance are arranged. Compared to translate the book, Turkey's textbooks 1. contains mostly algebraic expressions, does not use justifications and explanations, and is unrelated to real life, 2. contains applications that use more formulas, 3. The definition is less emphasized, 4. not using technology, 5. They do not clearly express the connections with other subject areas (Economy physics etc.). In general, it was seen that translated textbooks were mostly related to real life, equipped with explanations and justifications requiring cognitive competencies, and proceeded harmoniously between the subject area's main ideas and related ideas. These books use multi-step solved problems. Turkish textbooks need to be reviewed in terms of their functional areas in terms of context, cognitive need, representations, technology, and performance.

Keywords: mathematics textbooks, slope, Turkey, United States (Translated Turkish)

1. Introduction

We can not think without concepts. Slope is a critical topic based on its prominence throughout the all curriculum, the multitude of ways it can be conceptualized, and the role it plays in the development of more advanced mathematical ideas. A conceptual understanding of slope is the beginning of general mathematics. The prerequisite for mathematical analysis starts with the concept of analyzing slope and function in algebra information correct equations in school mathematics. Limit and derivative are reached on these concepts. The slope, direction, and steepness of an initial line is examined. Also, the slope of a line is defined as the ratio of vertical rise to horizontal progression as it passes from one point to another along the line. For example, when linear functions are represented algebraically, they can take the form $ax + by + c = 0$ or $y = mx + b$ or $y = mx +$ c with two unknowns. In these cases, the slope is represented as a ratio, in m or a $\frac{a}{b}$ or $\frac{\Delta y}{\Delta x}$, respectively. The focus of equations with an unknown is this rate of change. Examining how the perimeter and area of the square change physically are taken into consideration in the concept of function (Tuluk, 2007; Cooney, Beckmann, Lloyd, Wilson, & Zbiek, 2010). It can also be calculated as $\frac{y_2-y_1}{x_2-x_1}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$as an algebraic formula. Geometrically, it is the rate of $\frac{\text{vertical change}}{\text{horizontal change}}=\frac{rise}{run}$or it can indicate the slope of the road, expressed as a percentage. The angle trigonometrically made by a line with the positive direction of the x axis is $m = tan\theta$. However, when we consider different representations of functions, we move away from geometry. In general mathematics, $\text{l}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is used.

In mathematics, function comes to mind as a situation, an algebraic definition, table, graphic, or formula (Janvier, 1987). Regardless of the format, the rate of change of a function is the ratio of the change in the dependent variable to the change in the independent variable. This is the same ratio used to represent the slope. So, even though the slope has a reference in geometry, it is the rate of change and is therefore meaningful in terms of formula, table, physical state, and algebraic definition. Moreover, the slope is closely related to the concept of the derivative. The slope, therefore, contains many representations (eg geometrical, algebraic, trigonometric, and functional). It is a deep mathematical idea that is closely linked to the concept of function while advancing the limit and derivative in the curriculum.

Proportional reasoning is an important mathematical prerequisite for the concept of slope (Lamon, 1995). Children have difficulties with proportional reasoning (Hart, 1981; Heller et al., 1990; Karplus, Pulos, & Stage, 1983; Singer & Resnick, 1992; Thompson & Thompson, 1994, 1996). He argues that in authors such as Kaput and West (1994) and Thompson (1994), the rate is intuitive, but time can vary proportionally to another amount and is a minor structure for students as a quantity. Further abstraction is required to develop a ratio image that includes the equivalent of two non-constant quantities and the average rate of change of an amount in an independent quantity range. This abstraction can be difficult even for teacher candidates (Simon & Blume, 1994).

Findings show that propositions consist of hierarchical structures. Collins and Quillian (1969) have shown that people hide information at the highest level of generalization. For example, for an expert in the USB network for "function" (Even, 1993; Williams, 1998) in top-level cases such as "mapping-correlation-formula", "definition properties - types --- operation (limit, derivative, integral)" When registering, it is recorded differently for a student studying at the undergraduate.

Behavior (decreasing-increasing-derivative-critical points-slope-rate of change) comes additionally in categories for a student at the undergraduate degree. The information is stored in this way (Kertil, Erbaş, and Çetinkaya, 2017). Collins and Quillian (1969) find that the more distant the concepts are in memory, the greater the time they are brought back. The difficulty of the meaning of the slope becomes more difficult with various common meanings associated with the word "slope". It also has many concept images, such as a horizontal, vertical, ramp, or vertical, or causing slope or slope.

Rittle-Johnson, Siegler, and Alibali (2001) studied that the understanding that needs to be developed for conceptual knowledge about slope is knowledge of rules and performing certain tasks that can be transferred to various problems and situations regarding the process. The findings are that students do not address the slope with alternative representations or enrich and deepen the conceptual knowledge required to apply the meaning of slope in real-world situations (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Harel, Behr, Lesh, & Post, 1994; Lobato & Thanheiser, 2002; Reiken, 2009; Stump, 2001b; Teusher & Reys, 2010)

The use of real-world representations is thought to help students develop an understanding of abstract mathematics (Fennema & Franke, 1992). Researchs indicate the difficulties of students in understanding slope in both functional and physical situations (Bell & Janvier, 1981; Janvier, 1981; McDermott, Rosenquist, & Zee, 1987; Orton, 1984; Simon & Blume, 1994; Stump, 2001). Functional situations involving the slope are particularly important, highlighting the review of functions in high school with the latest recommendations (NCTM, 1989, 2000).

In particular, the researchers stated that students do not link the various impressions of the slope and do not relate the concepts of slope and rate of change (Stump, 2001b; Teuscher & Reys, 2010). It is seen that students have difficulty in using the slope in real-world applications (Lobato & Siebert, 2002; Lobato & Thanheiser, 2002).

Stump (2011) interviewed twenty-two high school students (before Calculus) using the concept of the slope using criteria. In the study, the students discussed real-world situations involving the slope, discussed, for example, physical states that include the slope as the measure of orthogonality, functional situations that include the slope as the measure of the rate of change. He found that for various physical situations, students measured steepness at angles rather than proportion. In general, they explained that they demonstrate that the slope is better understood in functional situations, but many students find it difficult to interpret the slope as a measure of the rate of change. Stump suggested that teaching focuses on providing students with connections and opportunities for students to improve their understanding.

The mathematical analysis begins by studying change, that is, creating the rate of change and ideas about slope. Both tutorials and textbooks have responsibilities in helping students understand this (Hill, Ball, Blunk, Goeyey, and Rowan, 2007). Today, a wide range of changes from education to the economy are used and life is in the areas of comprehensive learning.

Using researched examples in teaching and even using models are supported by researchers who have many theoretical tendencies. Cognitive modeling learning (cognitive skill learning, rule learning, motor skill learning) and the benefits of showing and explaining are known (Rosenthal & Zimmerman, 1978; Schunk, 1981). From a motivational point of view, the studied examples enable students to understand the sampled model and apply the given strategies on their own and strengthen students' sense of self-efficacy towards success (Schunk, 1991). The studied examples also have an important place in concept acquisition theories (Bruner, Goodnow Austin, 1956). The studied examples are the link between the principles of cognitive learning - especially principles in learning algebra, physics, and geometry (Atkinson, Derry, Renkl & Wortham, 2000; Atkinson, Renkl & Merrill, 2003). Here we come across the beginner-expert model application. The problem is that novices are found to be dealing with superficial features. While it is not much effective in skill development alone, it gives more effective results when applied with studied samples (Atkinson et al., 2000).

We can see the application of studied samples in the 4-stage skill acquisition model within the framework of ACT-R (Adaptive Control of Thought—Rational) (Anderson, Fincham, & Douglass, 1997). In the first stage, students establish similarities between the problems they need to solve with examples, in the second stage they develop abstract operational rules through repetition. In the third stage, as the problem-solving strategies start to automate, the student shows faster and fluent performance. When it comes to the fourth stage, many problem types are now placed in the student's memory. The student can recall the strategy suitable for the solution to the problem he confronts. The studied samples are most suitable for the 1st and 2nd stages. In later stages, students can advance their strategies through repetition. Nevertheless, studying examples of experts can benefit students even in the later stages.

Gagné believed that "the central point of education is to teach people to think, to use their rational powers, to be better problem solvers" (1980, p. 85). Like Gagné, most psychologists and educators see problem-solving as the most important learning outcome for life.

Textbooks have been related to teaching and learning as a teaching and learning instrument and they link teachers, students and the searched knowledge. Undergraduate mathematics students often read textbooks while doing their homework or to study for an exam and they do this primarily by looking at the solved examples (Weinberg, Wiesner, Benesh, & Boester, 2010). Problem-solving has been an important content domain in the mathematics curriculum. To make a topic problematic is to let students wonder, question, search for answers, and analyze and resolve inconsistencies. The study has taken the direction of analyzing the concept of slope, which is the beginning of the textbooks taught in higher education after the above studies, related to the slope. Both the curriculum and teaching should start with problems, dilemmas, and questions for students (Hiebert & Wearne, 1996).

2. Purpose of the Research

Due to the situations explained above, the concept of slope in mathematics textbooks has been considered and this situation has been directed to investigate the situation in problem-solving. Teaching situations in mathematics textbooks should be evaluated in terms of providing opportunities to encourage students to establish relationships and to think conceptually and in terms of educational goals and learning outcomes. These teaching situations are valuable in terms of revealing the nature, processes, and relationship systems (Stein, Grover, & Henningsen, 1996) of the underlying mathematical concepts for a better understanding of any mathematical thought. Thus, the mathematical situations presented in the textbooks help the student develop, reflect on, and make sense of mathematics (Stein et al., 1996). For this reason, it is also valuable to examine the mathematics textbooks used in higher education and to see how the content of those who will benefit from these textbooks is obtained.

For this purpose, this research aims to answer the following research questions: "What is the state of the slope related issues of solved problems in some textbooks used in undergraduate teaching in Turkey?"

3. Method

In the study, the document analysis method was used to analyze the slope in textbooks. Document analysis is the process of collecting the existing records and documents related to the subject to be investigated and encoding them according to the specific norm or system (Cohen & Manion, 1994; Çepni, 2012).

3.1 Selection and Features of Textbooks

The developments in technology and economy add a different dimension to getting mathematics knowledge and the actions to reach it from books. The new century in which we live makes it imperative that we tend to gain glances towards sustaining mathematical knowledge through real-life and throughout life. In this study, two books written abroad and translated into Turkish and three books from Turkey used in undergraduate education were selected. In the selection of the books, the availability of the book, and its use in undergraduate education are taken into consideration.

While problems differ in nature, in the way they are presented or represented, their components, and their interactions, there is an agreement that they differ in content, structure, process (Jonassen, 2000). Although the experience of the student who used it in the textbook is other powerful factors such as the teacher who chose this book, the form of the textbook is important (Otte, 1986) and its complexity and interactions should be taken into account in terms of its problems.

Mathematics textbooks translated into Turkish have been selected for their undergraduate readiness towards mathematics lesson, taking into account the readiness to advance the limit and subsequent derivative issues through real life models, taking into account readiness for algebra from high school. All three of the textbooks written in Turkish proceed with the classical lecture that is still used in our country due to their approach. The slope is the initial concept of mathematical analysis.

Table 1. Solved Problems in Textbooks Analyzed

Textbook		Solved Problems	Page
TB1	Basic Functions Vocabulary	55	1-129
	Functions and Graphics		
TB2	Functions and Graphics	19	1-81
	Tangent Lines and Slope Estimators		
TuTB1	Functions	14	1-207
	Identities, equations, graphics		
	First and second-degree inequalities		
	Equation and inequality systems		
TuTB2	Analytical Analysis of the Line	9	50-58
TuTB3	Analytical Analysis of the Line	11	29-37

3.2 Analysis Criteria

This study, which dominates the presentation of the slope concept in the translation and interpretation of mathematics textbooks taught in undergraduate teaching in Turkey approaches for context based on previous studies (Rezat, 2006) in the context inspired by the work, connectivity, explore, and aim for cognitive requirement (Stump, 1999; 2001b) and Moore-Russo et al., 2011), geometric ratio, behavioral indicator, feature determinant, algebraic ratio, parametric coefficient, functional feature, linear constant, real-life, physical feature, trigonometric concept, calculus. Representations of process skills, algebraic expression, table, graphics are handled. Using technology (Akkoyunlu, 2002; Schware and Jaramillo, 1998) BCS, Scientific and graphic Calculators, definition, justification, the explanation for performance were taken into consideration (see Table 2).

Table 2. Criteria Used in the Analysis of Textbooks

Criteria Used in the Analysis of Textbooks	
1. Context	Connectivity (C)
	Purpose (P)
	Exploring (EX)
2. Cognitive requirement	Geometric ratio (GR)
	Behavior indicator (BI)
	Determining property (DB)
	Algebraic ratio (AR)
	Parametric coefficient (PC)
	Functional property (FP)
	Linear constant (LC)
	Real-world situation (RWS)
	Physical property (FP)
	Trigonometric conception (TC)
3. Representation	Table (T)
	Graphic (G)
	Algebraic expression (AE)
4. Technology	Computer algebra system (CAS)
	Scientific and graphic Calculators (SGC)
5. Performing an action, task or function (performance)	Definition (D)
	Explaining (E)
	Justification (J)

3.3 Sample Codings

The question in TuTB1, 1. Write the equation of the line L, which passes through the point $P\ (3{,}5)$ and is parallel to the line $y = 2x - 4$.

In this problematic analysis, EX = 1, BI = 1, DB = 1, PC = 1, LC = 1, FP = 1, G = 1, CI = 1, D = 1, E = 1, J = 1.

The question in TB2, Cost equation: A skateboarding company management has a fixed cost of $ 300 per day (when production is 0) and a total cost of $ 4300 per day when 100 skateboards are produced daily. Suppose that there is a linear relationship between x and x, as shown by cost, production quantity.

a. Find the equation of the line passing through the points corresponding to 0 and 100 production quantities. More specifically, what is the slope of the line passing through points (0, 300) and (100, 400)?

b. Find the linear equation expressing the cost depending on the amount of production. Write the result in the form.

c. Plot the cost equation above.

In the last explanation, this question ends with the fact that when the production amount rises to 10, the variable number 40, which is the slope of the line, can be interpreted as the "rate of change" of the cost function depending on the production amount.

In analyzing this problematic solution, C = 1, P = 1, EX = 1, GR = 1, BI = 1, DB = 1, AR = 1, PJ = 1, LC = 1, RWS = 1, KK = 1, G = 1,AE = 1, D = 1, E = 1, J = 1.

In the problem-solving problem assessment in TuTB1, EX = 1, PC = 1, DB = 1, LC = 1, AE = 1, D = 1, E = 1.

In the problem-solving problem assessment in TuTB2, EX = 1, PC = 1, DB = 1, LC = 1, AE = 1, D = 1, E = 1.

In the problem-solving problem assessment in TuTB3, EX = 1, PC = 1, DB = 1, LC = 1, AE = 1, D = 1, E = 1.

4. Results

In the translated textbooks used in this study, in terms of context, the solved problems related to slope tend to be 25% connectivity, 41% purpose, and 73% discovery, while the percentage of problems in three Turkish books is 1%. In the context of the slope, translation textbooks in the undergraduate language differ from Turkish textbooks in terms of connectivity, purpose, and discovery.

Table 3. Criteria Regarding Solved Problems in Analyzed Textbooks

Criteria Regarding Solved Problems in Analyzed Textbooks			
Context	Connectivity (C)	%25	%0
	Purpose (A)	%41	%0
	Discovery (D)	%73	%1
Cognitive requirement	Geometric ratio (GR)	%30	%1
	Behavior indicator (BI)	%40	%33
	Determining property (DB)	%3	%9
	Algebraic ratio (AR)	%32	%23
	Parametric coefficient (PC)	%32	%30
	Functional property (FP)	%23	%0
	Linear constant (LC)	%48	%2
	Real-world situation (RWS)	%35	%0
	Physical property (FP)	%41	%1
	Trigonometric conception (TC)	%10	%4
Representation	Table (T)	%2	%0
	Graphic (G)	%32	%20
	Algebraic expression (AE)	%,100	%100
Technology	Computer algebra system (BCS)	%1	%0
	Scientific and graphic Calculators (SGC)	%7	%0
Performing an action, task or function (performance)	Definition (D)	%80	%4
	Explaining (E)	%52	%8
	Justification (J)	%48	%8

In the translated textbooks used in this study, in terms of cognitive requirement, the problems solved with the slope are 30% geometrical ratio, 40% behavioral indicator, 3% determining property, 32% algebraic ratio, 32% parametric coefficient, 23% functional feature, 48% linear constant, 35% real life, 41% physical property, 10% trigonometric concept. In the three Turkish books used in this study, the percentage of the problem for the geometric rate in cognitive need is 1%, 33% is the behavioral indicator, 93% is the determining property, 23% is the algebraic rate, 30% is the parametric factor, 0% is the functional property, 2% linear constant, 0% real life, 1% physical property, 4% trigonometric concept.

The problems solved with the slope in the translated textbooks used in this study are 2% for the table in representation, 32% for graphical representation, and 100% for algebraic expression. In the three Turkish textbooks used in this study, the problems solved with the slope are 0% for the table in representation, 20% for graphical representation, and 100% for algebraic expression.

In the translated textbooks used in this study, the problems solved with the slope are 1% for BCS in technology use and 7% in scientific and graphic calculator use. In the three Turkish textbooks used in this study, the problems solved with the slope are 0% for BCS in technology use and 0% in scientific and graphic calculator use.

In the translated textbooks used in this study, the problems solved with the slope are 100% for definition, 52% for the explanation, and 48% for justification. In the three Turkish books used in this study, the problems solved with the slope are 4% for definition, 8% for the explanation, and 8% for justification.

In the translation textbooks used in this study, the solved problems related to slope tend to be 25% connectivity, 41% aiming and 73%, while the percentage of problems in three Turkish books is 1%. In the context of the slope, translation textbooks in the undergraduate language differ from Turkish textbooks in terms of purpose, connection, and discovery.

In the translated textbooks used in this study, the problems solved with the slope are 53% geometrical ratio, 62% behavioral indicator, 6% feature determiner, 64% algebraic ratio, 83% parametric coefficient, 52% functional feature, 92% linear constant, 35% real life, 32% physical property, 29% trigonometric concept.

In the three Turkish books used in this study, the percentage of the problem for the geometric rate in cognitive need is 0.7%, 33% is the behavioral indicator, 93% is the property determiner, 43% is the algebraic ratio, 100% is the parametric factor, 0% is the functional property, It is 0% linear constant, 0% real life, 1% physical property, 41% trigonometric concept. In this study, undergraduate translation textbooks for problems solved with slope differ from Turkish textbooks in cognitive requirements.

The problems solved with the slope in the translation textbooks used in this study are 0% for the table in representation, 32% for graphical representation, and 42% for algebraic expression. In the three Turkish books used in this study, the problems solved with the slope are 2% for the table in representation, 20% for graphical representation, and 100% for algebraic expression. In this study, undergraduate translation textbooks for problems solved with slope differ in using representation from Turkish textbooks. The table is not used in the handling of the concept of equation and function in Turkish textbooks. In graphic representation, the situation is in favor of translation textbooks.

In the translation textbooks used in this study, the problems solved with the slope are 1% for BCS in technology use and 7% in scientific and graphic calculator use. In the three Turkish books used in this study, the problems solved with the slope are 0% for BCS in technology use and 0% in scientific and graphic calculator use. In this study, undergraduate translation textbooks for problems solved with slope differ from Turkish textbooks in using technology. There is no information regarding the use of BCS or scientific and graphic calculators to deal with the concept of equation and function in Turkish textbooks.

In the translation textbooks used in this study, the problems solved with the slope are 100% for definition, 52% for the explanation, and 48% for justification. In the three Turkish books used in this study, the problems solved with the slope are 4% for definition, 8% for the explanation, and 8% for justification.

5. Discussion and Conclusion

Students tend to use reading strategies that are completely different from those used by expert readers. The studied examples are more useful for students who are in the early stages of skill acquisition rather than advanced students trying to perfect their skills. Undergraduate math students often read textbooks while doing their homework or to study for an exam and they do this primarily by looking at the solved examples (Weinberg, Wiesner, Benesh, & Boester, 2010). Students rely on completing homework problems, "identifying similar superficial features in a sample, some other situation previously described in a theorem, rule, or text" and "imitating the procedure in the described scenario." (Lithner, 2003).

Textbooks play an important role in undergraduate mathematics courses and have the potential to affect student learning (Weinberg et al., 2012). However, few studies describe students' use of the textbook in detail.

Understanding the concept of slope in a context stands for the use of real-life in the slope problems in the translated and Turkish textbooks used in this study.

It stands in favor of table and graphic translated books in terms of representation of slope problems in the translated and Turkish textbooks used in this study. Creating and using representations for organizing, recording, and communicating mathematical ideas, choosing mathematical representations to solve problems, and switching between them, using representations to interpret and model physical, social and mathematical events are among the process skills of mathematics (Wan De Walle, 2013, p. 4). Technology provides many opportunities for discussion. The same is true for the graphics. Graphs are tools that facilitate data editing, interpretation, and presentation; It helps us to see the details while summarizing a lot of data. At the same time, graphs are not only easy to express mathematical relationships with numbers but also are helpful tools for solving arithmetic and algebraic problems and expressing complex relationships between variables and help students develop concepts (Beichner, 1994; Ersoy, 2004; McKenzie & Padilla, 1983; 1984).

The use of the definition in the performance requirements of the slope problems in the translated and Turkish textbooks used in this study stands in favor of the translated books. Many students only have procedural knowledge of algorithms for slope and interpretation of slope in certain situations. These are expressions such as finding the slope of a line passing through two given points, finding the slope of a given line, finding the slope of a straight (vertical) parallel to a line, or taking into account the ratio of vertical and horizontal displacement of a line. This is how the subject is handled in Turkish textbooks.

Kertil, Erbaş, Çetinkaya (2017) explained how students think about rates of change in different inactive contexts (ie "marginal cost in economy", "flow rate in physics") in terms of pedagogical principles. They explained that more studies are needed in order to define the possible ways of effective thinking and development methods in students. They also proposed more research to clarify the semantic aspects of the rate of change in the same research. Such problems are rarely used in Turkish mathematics textbooks. It is necessary to reconsider the slope and the rate of change associated with Turkish textbooks. In the problems solved by slope in translated textbooks, a goal has been determined, ie the rate of change.

These results raise questions about the conceptual view of textbooks. The role of the authors regarding the definition, explanation and justification of the concepts to students is valuable. Enrichment of textbooks with teaching about problem solving with examples that question real life are important gains related to students' conceptual view. Handling computers and software in mathematics teaching at all levels provides an opportunity to broaden students' perspective. A mathematics textbook should give students the opportunity to link carefully selected contextual tasks between students' mathematics and practices and to see how mathematics can help gain meaning (Clarke, Rouche, 2018).

The nature of mathematics can be studied in the light of philosophical and pedagogical forms of teaching and learning. 20th century Based on the principle of non-falsification at the beginning, 3 paradigms dominate. Logicists, Symbolists, and Intuitioners. While these three traditions think that mathematics is precise, irreparable, universal, and abstract, a movement that argues that mathematics is built by falsifiable, applied, social, and individuals confront these three traditions. This movement is called quasi-experimentalism.

According to anthropological theory, there are two different types of knowing or knowing an object or an object in general terms: Individual Recognition and Institutional Recognition. In anthropological theory, learning is expressed as the change of the individual recognition of an X person belonging to the Object (O). This change means that if there is no individual recognition, it starts to exist and if it does, it means development. This learning changes the knowledge of the individual, not the individual. To talk about learning in anthropological theory, institutional definition, which is the fifth building block of this theory, should be defined primarily. This definition can start with the analysis of the books read. We can roughly summarize the sciences as formal sciences (mathematics and logic), physical sciences (physics and chemistry), life sciences (biology), social sciences (psychology and sociology), earth, and sky sciences (astronomy and earth sciences). It is the aim of the study to analyze the concept of slope that exists in these classifications by determining the institutional definitions for learning and teaching conditions at the university level through books. Then, the individual definitions developed by the students should be determined. Then a comparison of these two situations should come.

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Appendices

Appendix A List of textbooks used in the study Turkish textbooks:

TuTB1 Çelik, B., Cangül, İ. N., Çelik, N., Bizim, O., & Öztürk, M. (2013). Temel Matematik, Dora Basım Yayın LTd. ŞTİ. Bursa.

TuTB2 Aktaş, M. V. S. (2004). *Genel Matematik 1, Pegem A Yayıncılık, Ankara*.

TuTB3 Balcı, M. (2014). *Genel Matematik -1, Sürat Üniversite Yayınları.* İzmir.

U.S. textbooks (Translation):

TB1 Edwards, C. H., & Penney, D. E. (2001). *Matematik Analiz ve Analitik Geometri*, Palme Yayın Dağıtım Pazarlama İç ve Dış Ticaret A.Ş. Ankara. Çeviri Editörü Ömer Akın.

TB2 Barnett, M. A., Ziegler, M. R., & Byleen, K. E. (2017). *İşletme, İktisat, Yaşam Bilimleri ve Sosyal Bilimler İçin Genel Matematik* Çeviri Editörü: Prof. Dr. Arif Sabuncuoğlu Nobel Akademik Yayıncılık Eğitim Danışmanlık Tic. Ltd. Şti.yayın.com